Experimental Observation of Oceanic Forced Convection

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Parameterizations for global dynamical models in Climatology, Astrophysics and Planetology

LABORATOIRE DES ECOULEMENTS GEOPHYSIQUES ET INDUSTRIELS

Institut des géosciences de l'environnement

Outline

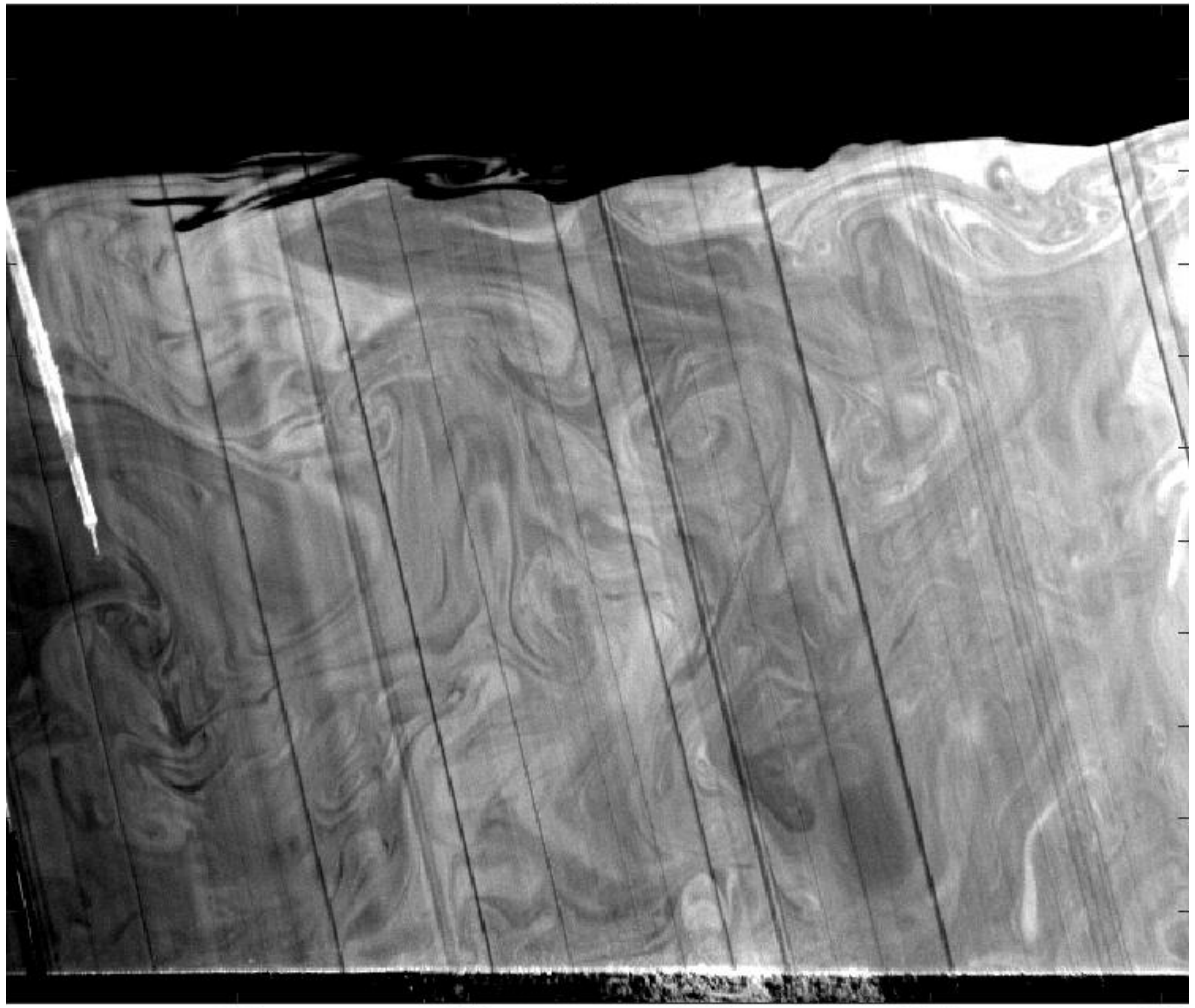
1. Introduction
2. Wind-driven ML deepening in ocean
3. Forced convection experiment
4. Longer time behavior of the ML
5. Conclusion

Credit : Facebook/Rachel Gordon

Introduction: Definition of Convection

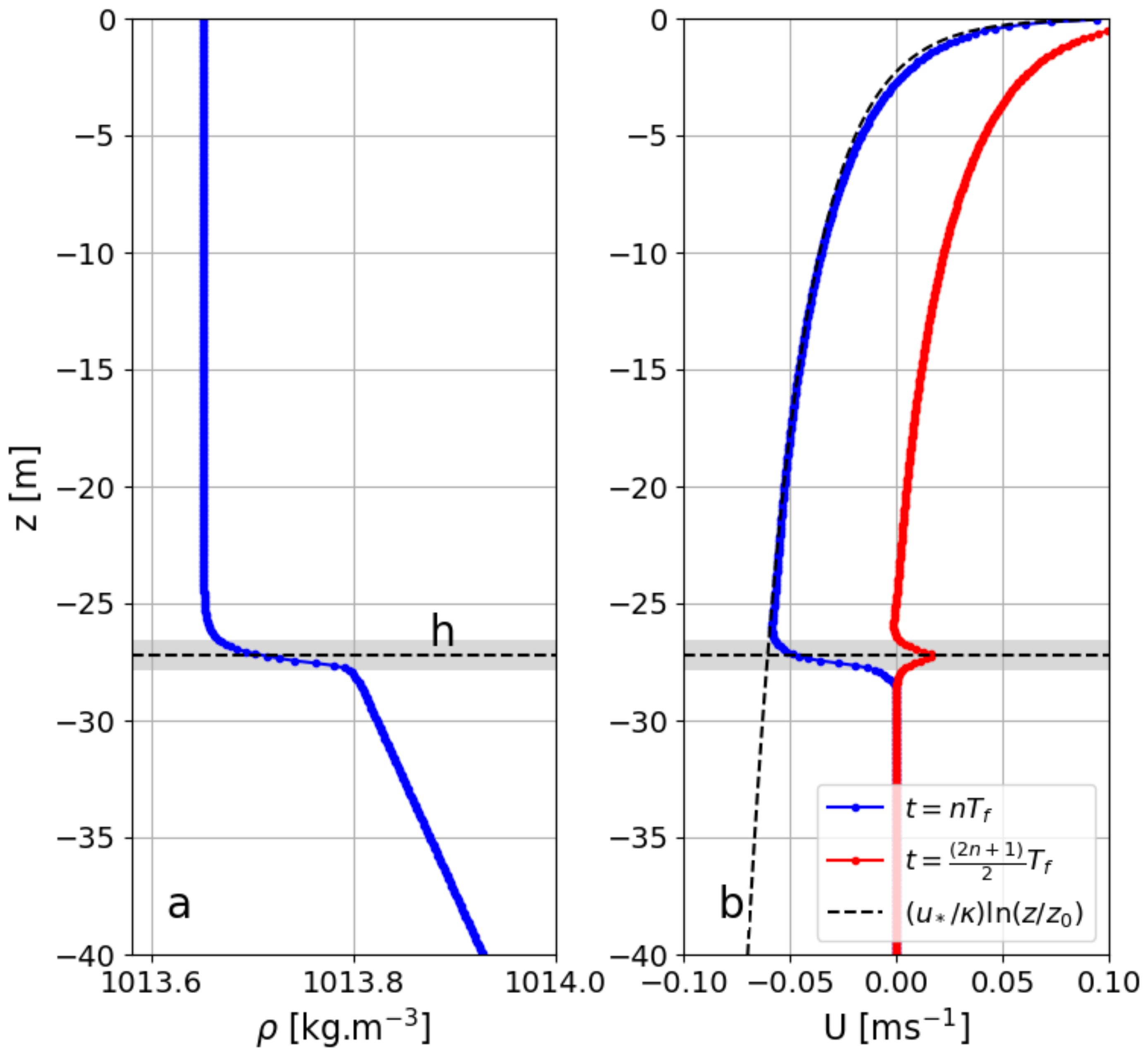
Vertical motion of a fluid parcel caused by:

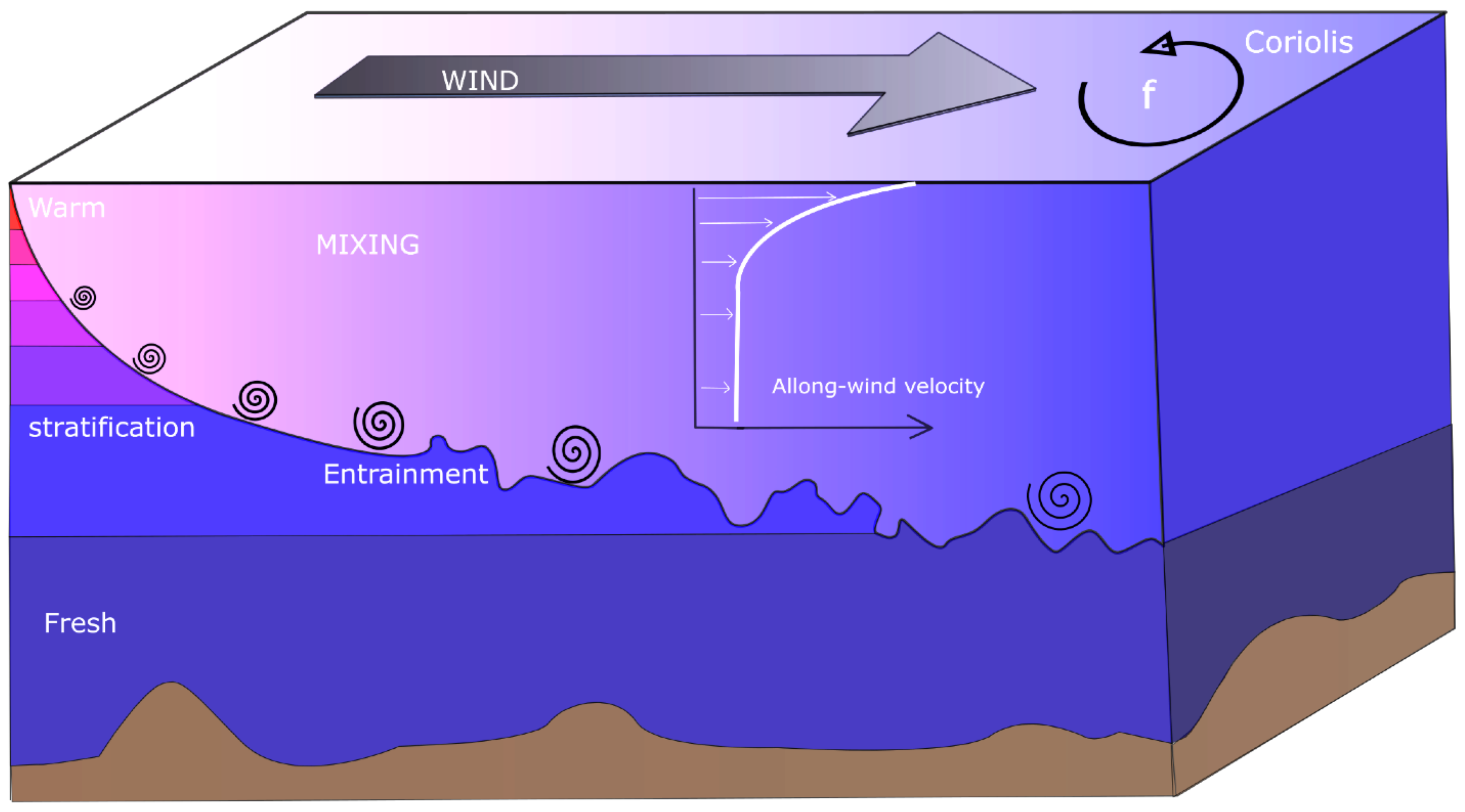
- Buoyancy Force → Free convection
- External Force → Forced convection



Wind-driven Oceanic Convection : Processes

Shear stress at the surface





Forced Convection (Momentum Flux)

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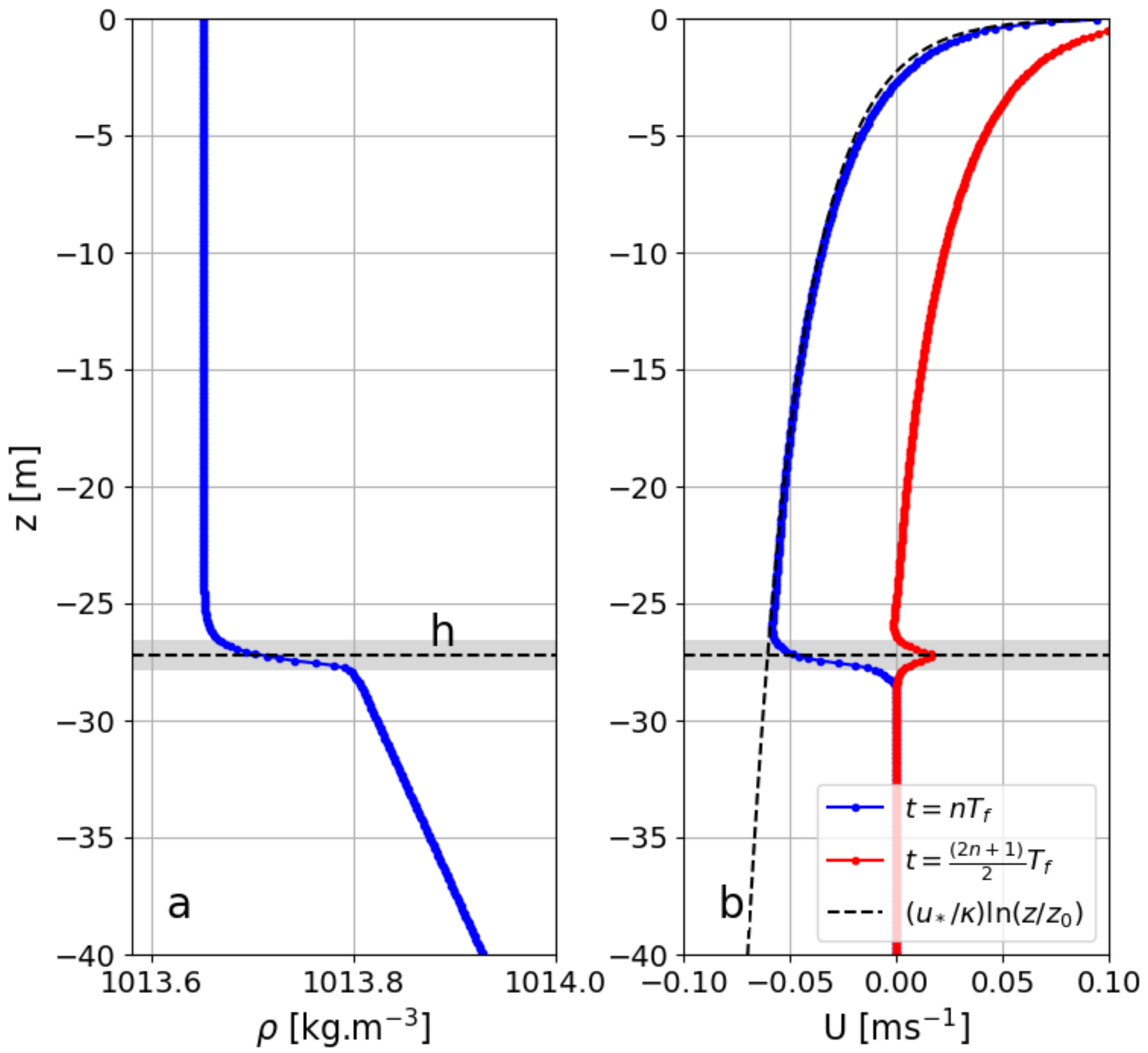
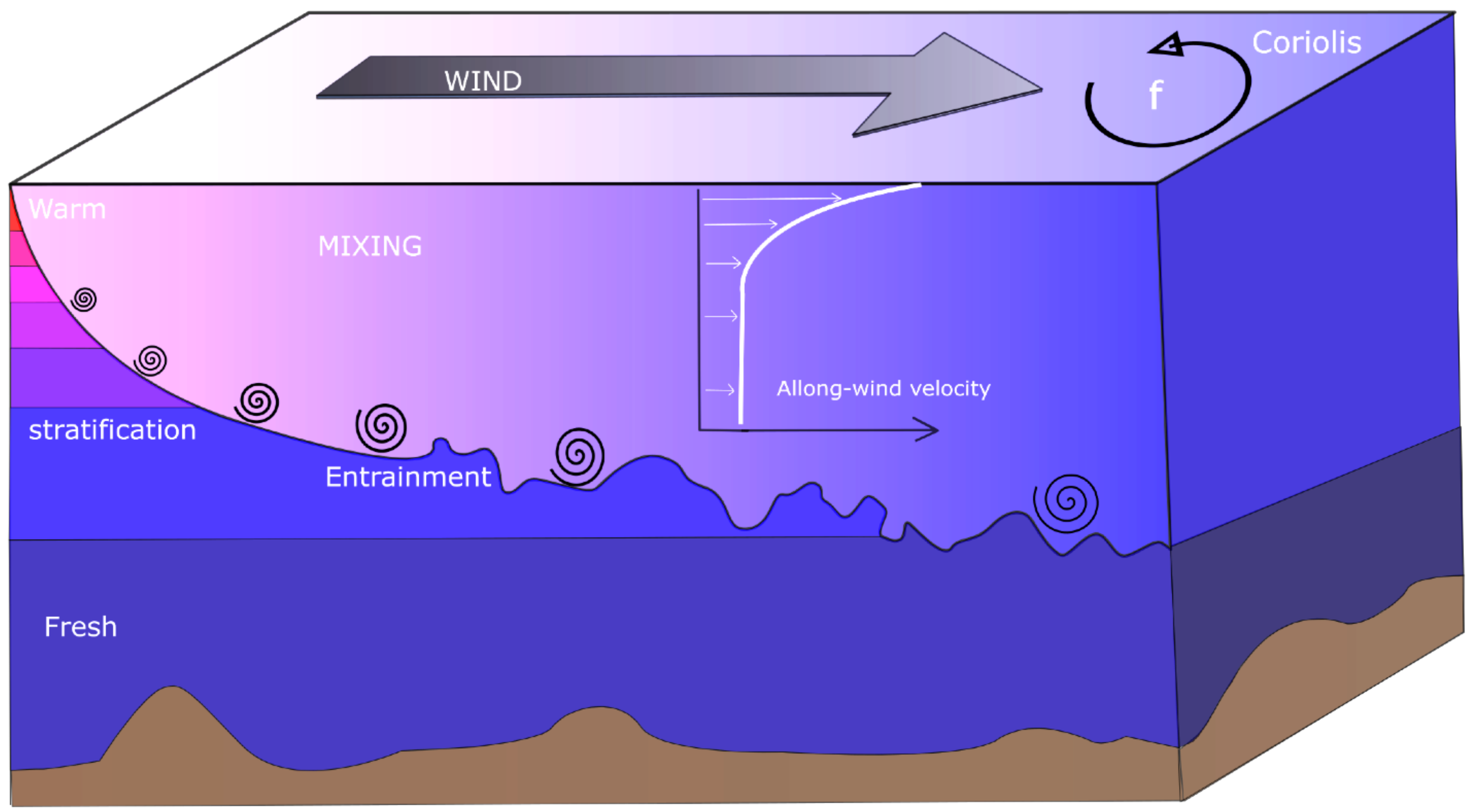
Log layer / Ekman Layer





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Wind-driven Oceanic Convection : Processes

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↓

Log layer / Ekman Layer

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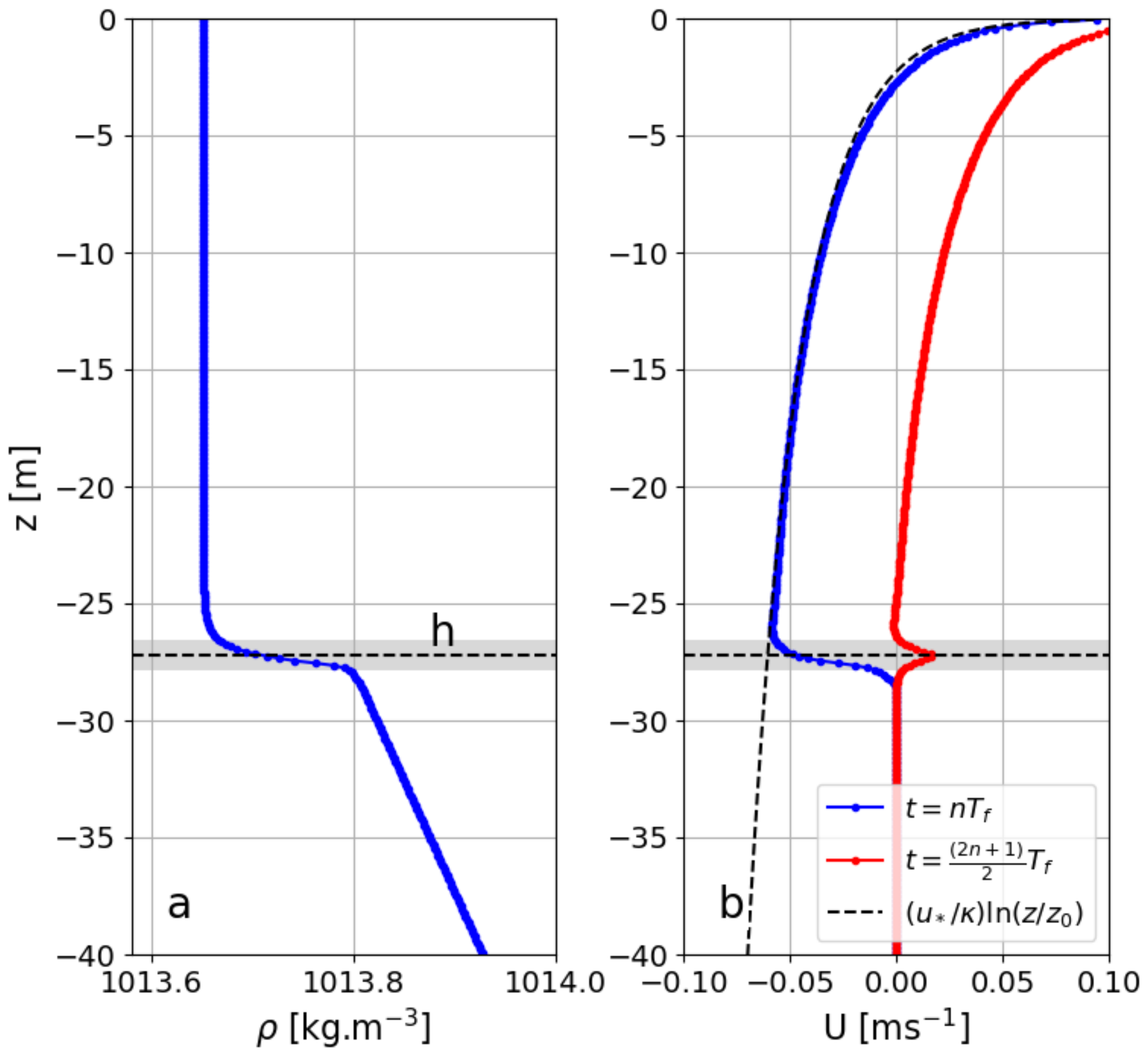
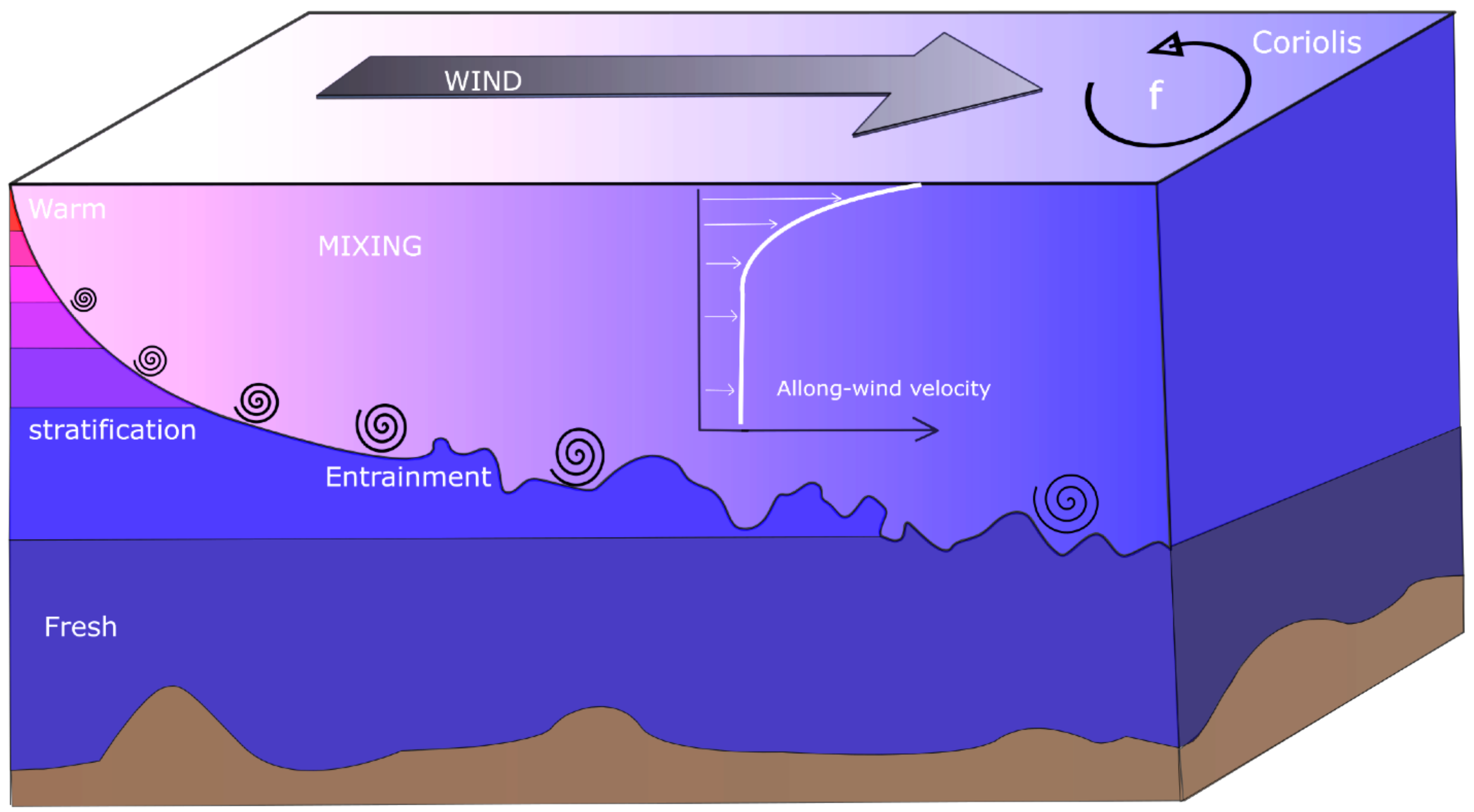
Entrainement turbulence





Forced Convection (Momentum Flux)

Wind-driven Oceanic Convection : Processes

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↓

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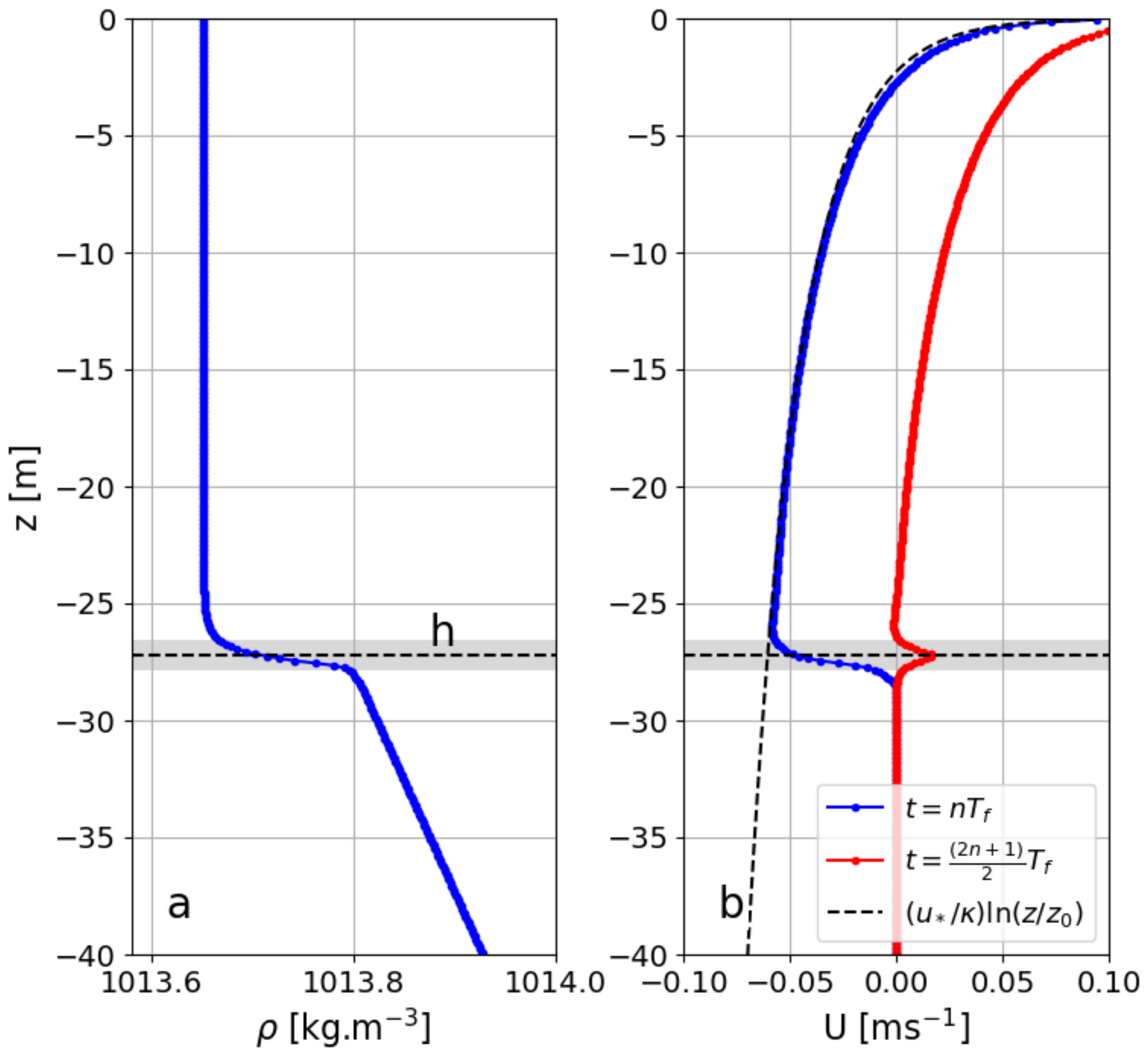
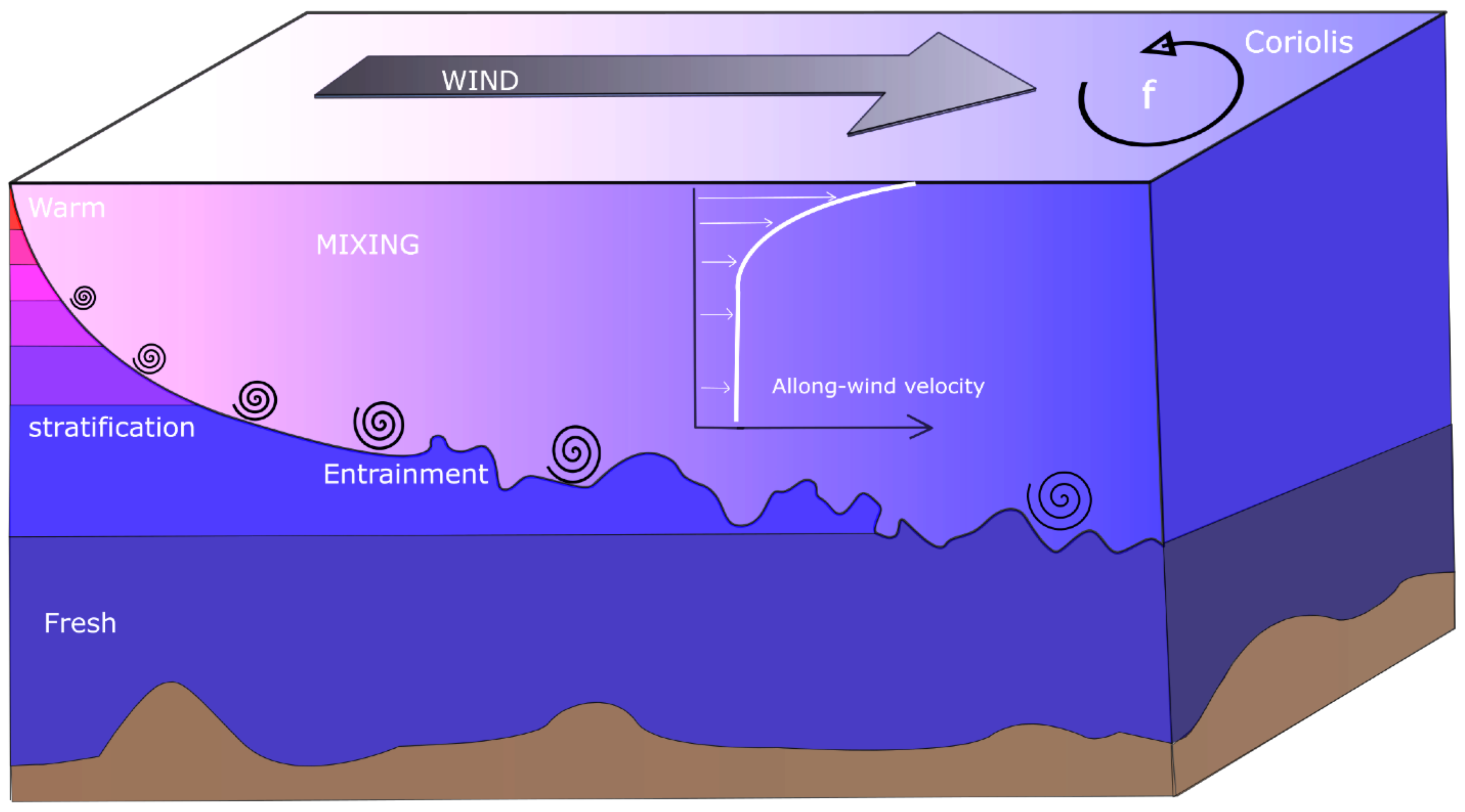
↓

Erosion of the Stratification





Forced Convection (Momentum Flux)

Wind-driven Oceanic Convection: Scaling law

Pollard et al, 1973: Assumption

- Linear stratification
- Bulk consideration
- The entire stress at the bottom of ML deepen it
- Excess of energy by wind + rate of change of TKE + Dissipation = 0

$$u_{surface} > u_{bulk}$$

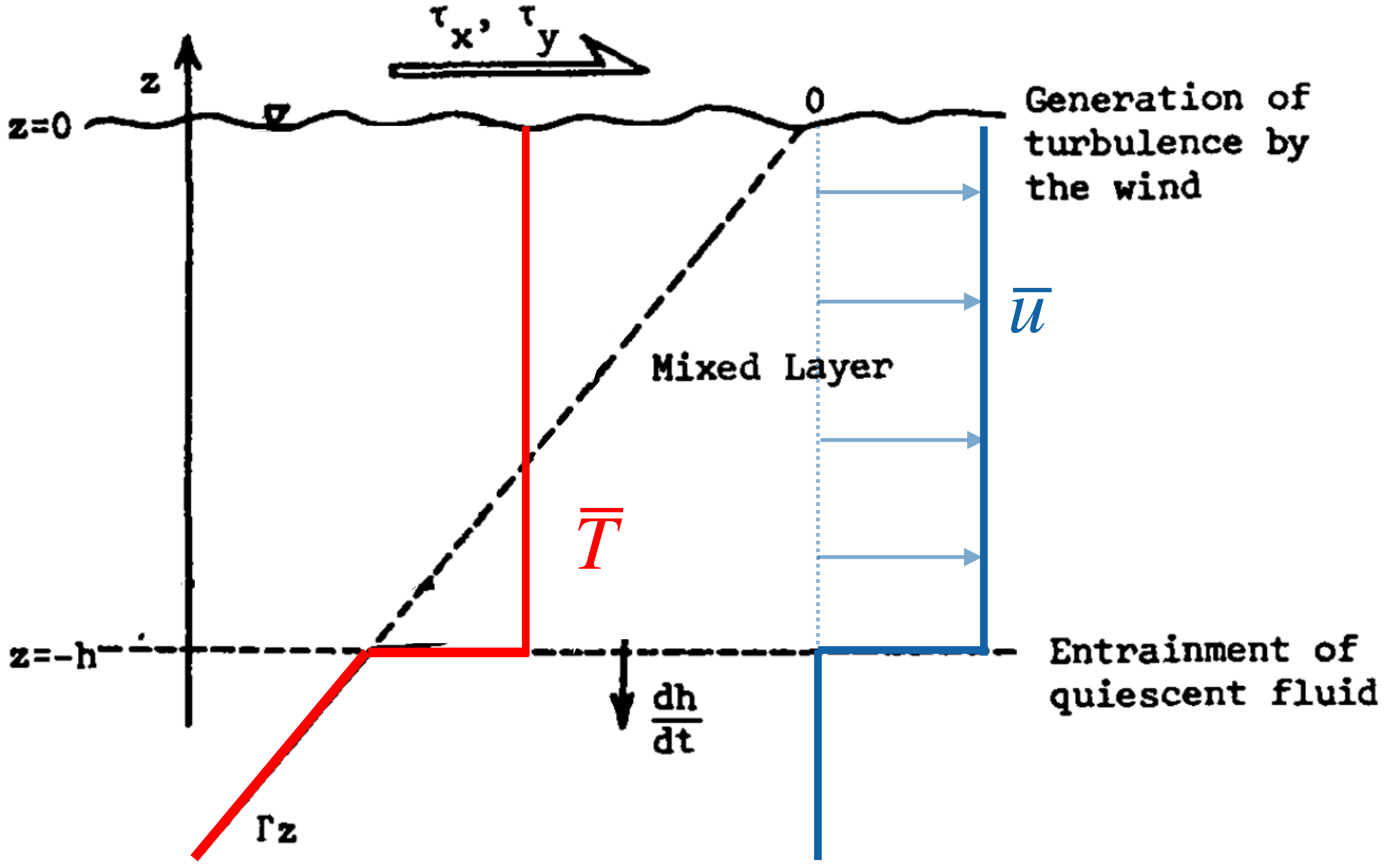


Figure modified from Cushman-Roisin, 1981

Pollard, R.T., Rhines, P.B., Thompson, R., 1973. The deepening of the wind-Mixed layer. Geophysical Fluid Dynamics.

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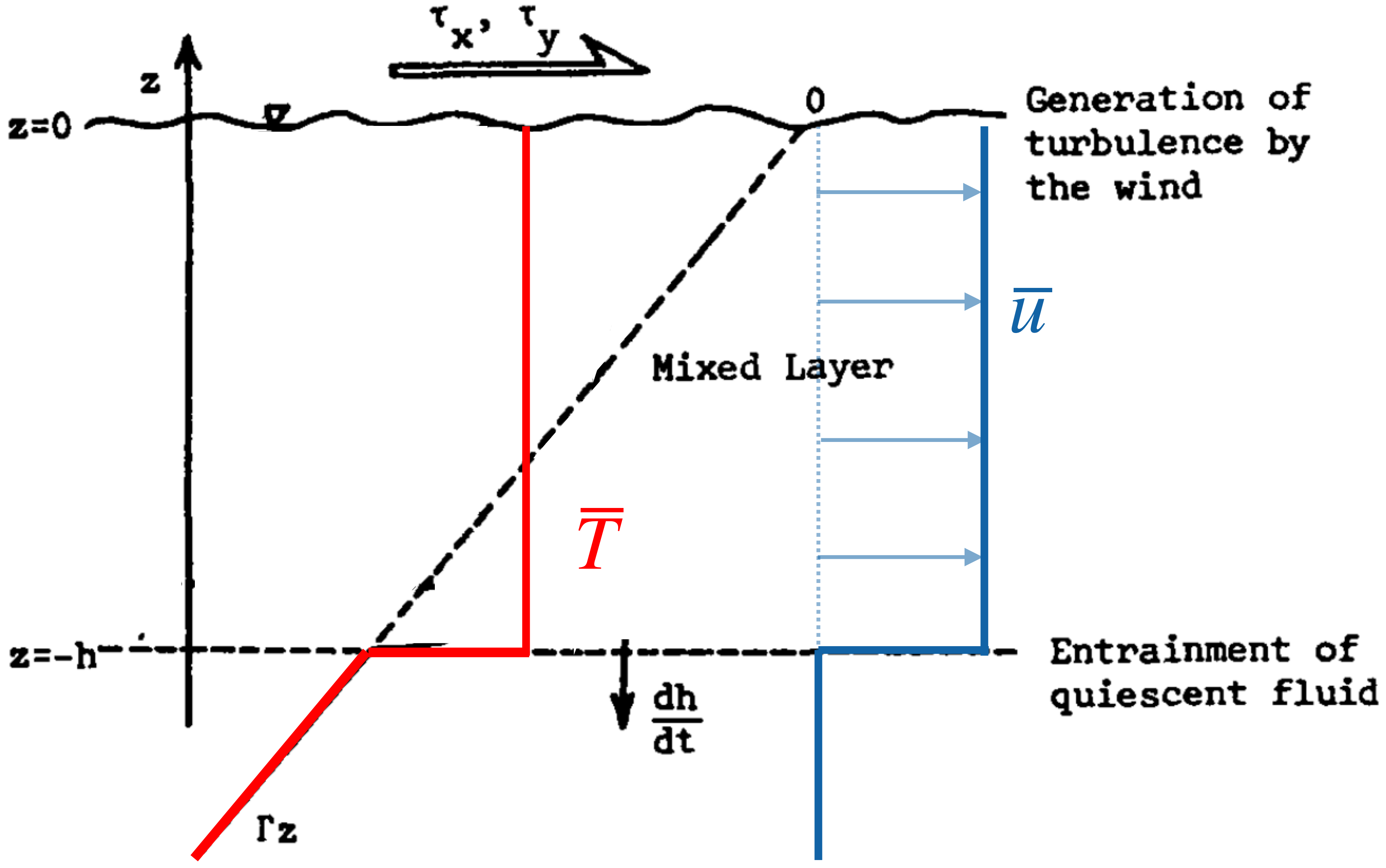
ML deepening scaling law

- Momentum of mixed Layer increase : $uh \propto u_*^2 t$
- Marginal stability state $\Rightarrow$ $Ri = \frac{h^2 N_0^2}{u^2} = Ri_c$

$$h_{(t)} \sim 2^{1/4} u_* \sqrt{\frac{t}{N_0}}$$

$$h_{(t)} = \frac{u_*}{\sqrt{N_0 f}} \left[4(1 - \cos(ft)\right]^{1/4}$$



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Seminal Experiments

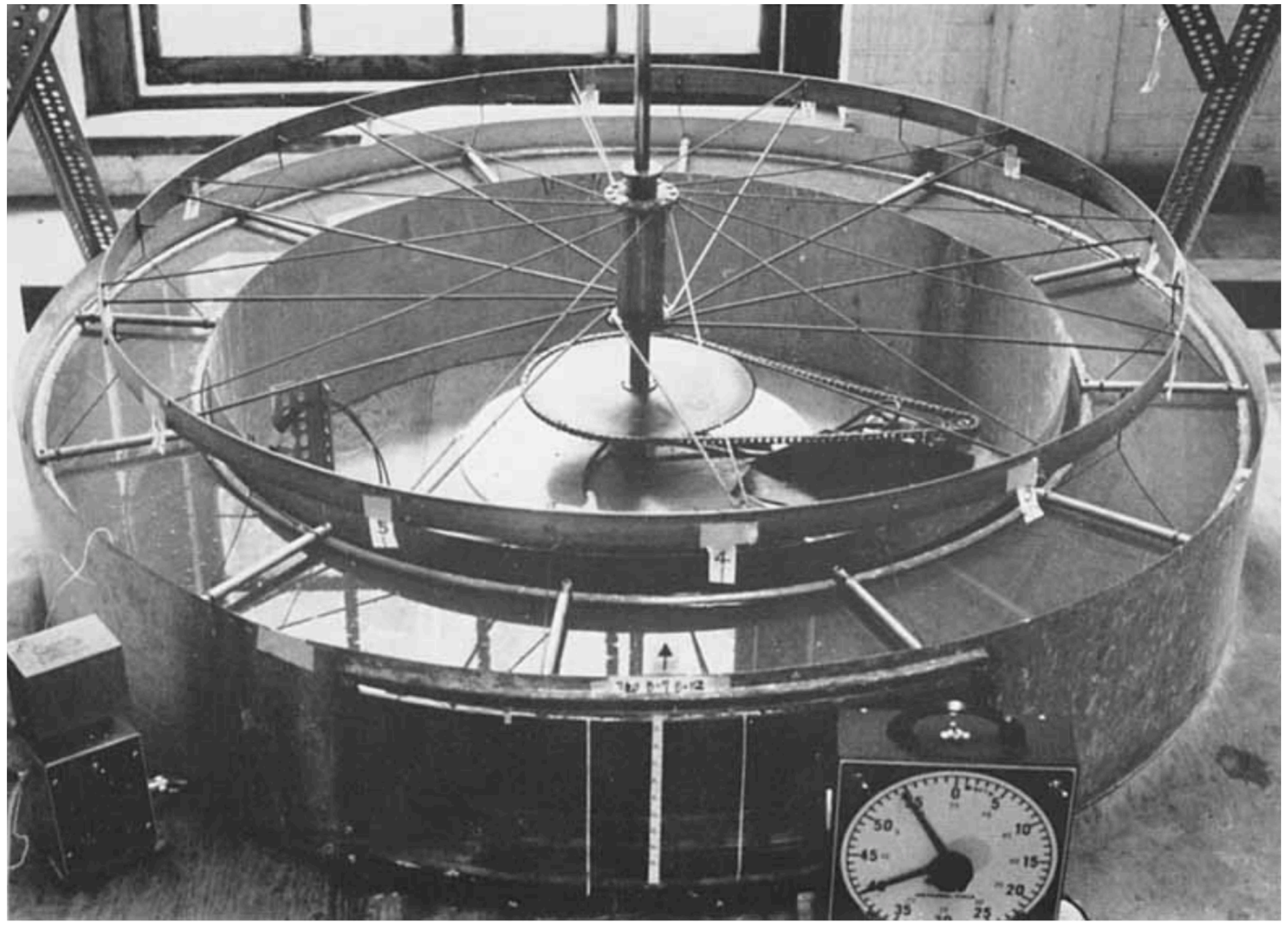


FIGURE 1. The experimental apparatus.

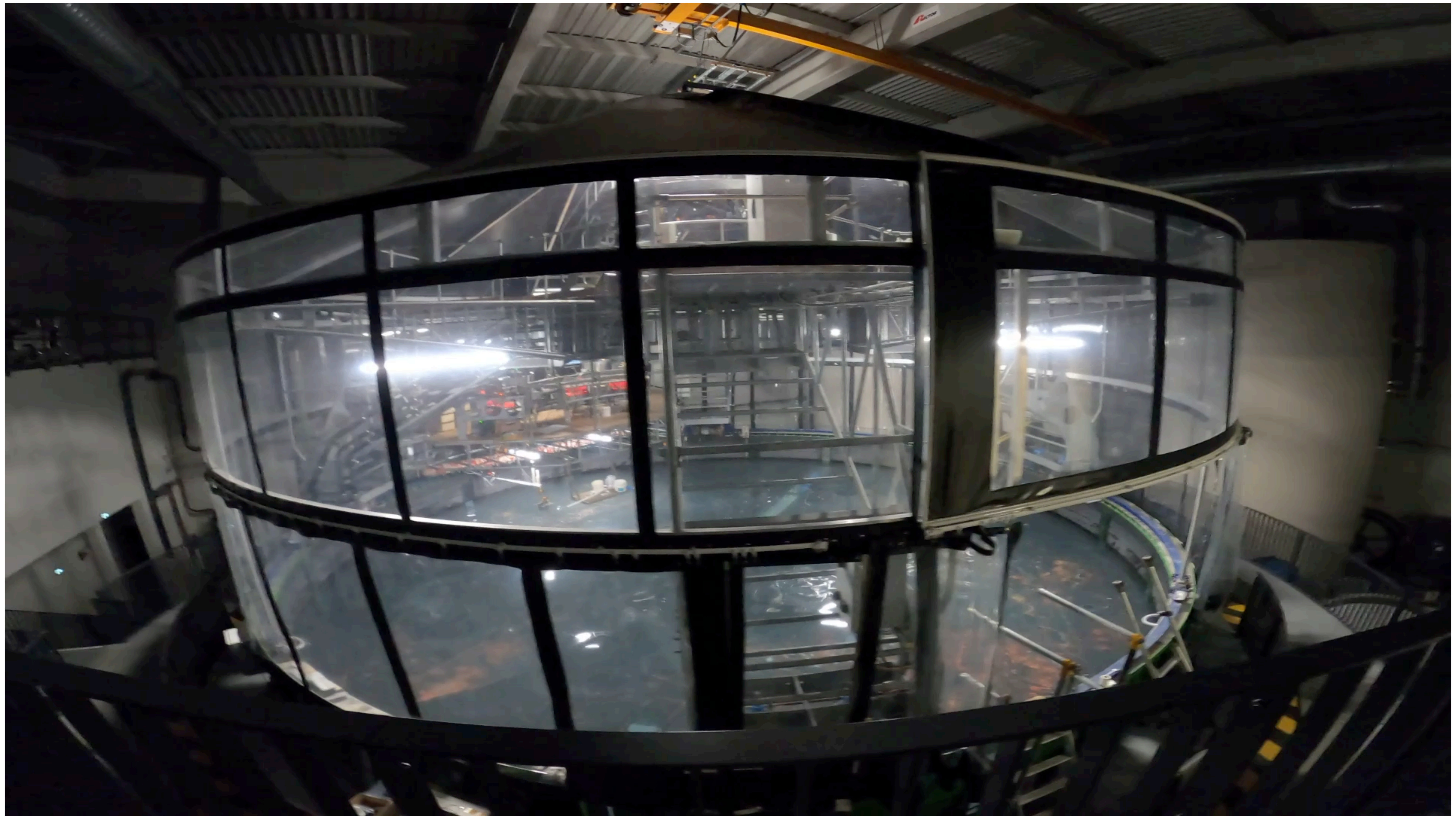
Kato - Philips 1969

- Mixed layer deepening rate : $h \sim t^{1/3}$
- Entrainment law : $E(Ri) = \dfrac{dh/dt}{u_*} = 2.5\, Ri^{-1}$

- They did not considered rotational effects
- The torque applied was tuned « by hand »
- No direct measurement of the density
- No direct measurement of the velocity

Kato, H., Phillips, O.M., 1969. On the penetration of a turbulent layer into stratified fluid. Journal of Fluid Mechanics 37, 643–655

Coriolis Platform

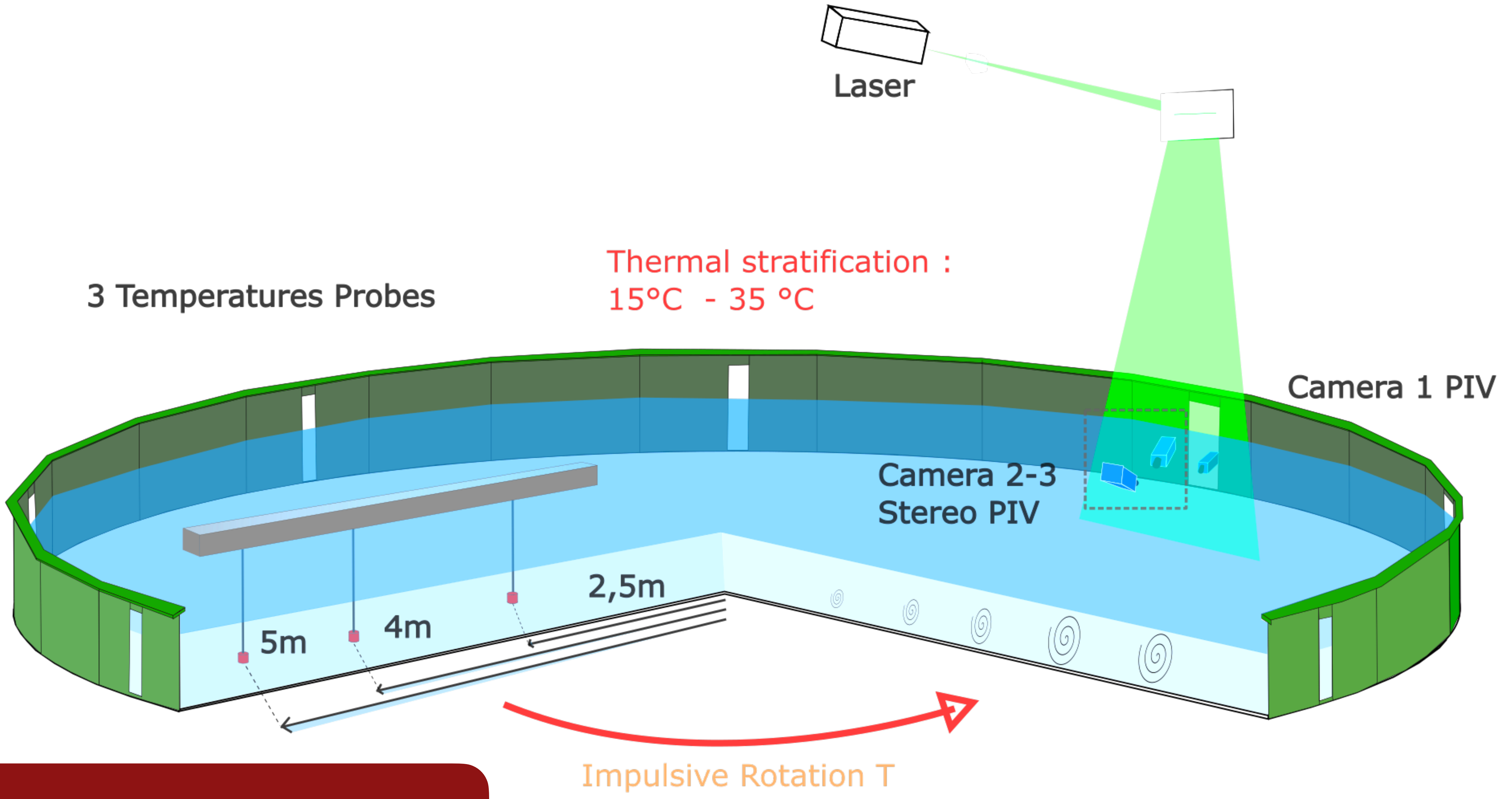


- **Diameter:** 13 m
- **Weight :** 350 Tones at full load
- **Maximum Speed:** 6 rpm
- **Max water height:** 1 m
- **Volume:** 132 m^3

- **Rossby Number** $Ro = \frac{U}{fL}$
- **Froude Number :** $Fr = \frac{U}{NL}$
- **Reynold Number:** $Re = \frac{UL}{\nu}$

Forced Convection Experiment: Apparatus

- **Acceleration of rotation (Spin-Up)**
- **Temperature stratification**
- **Temperature probes**
  - 3 Vertical profilers
- **Vertical laser sheet (30x25)cm**
  - PIV Stereo (2D - 3 components)



Control parameters

Friction : u_* **Rotation :** f **Stratification** $N^2 \equiv (\Delta T)$

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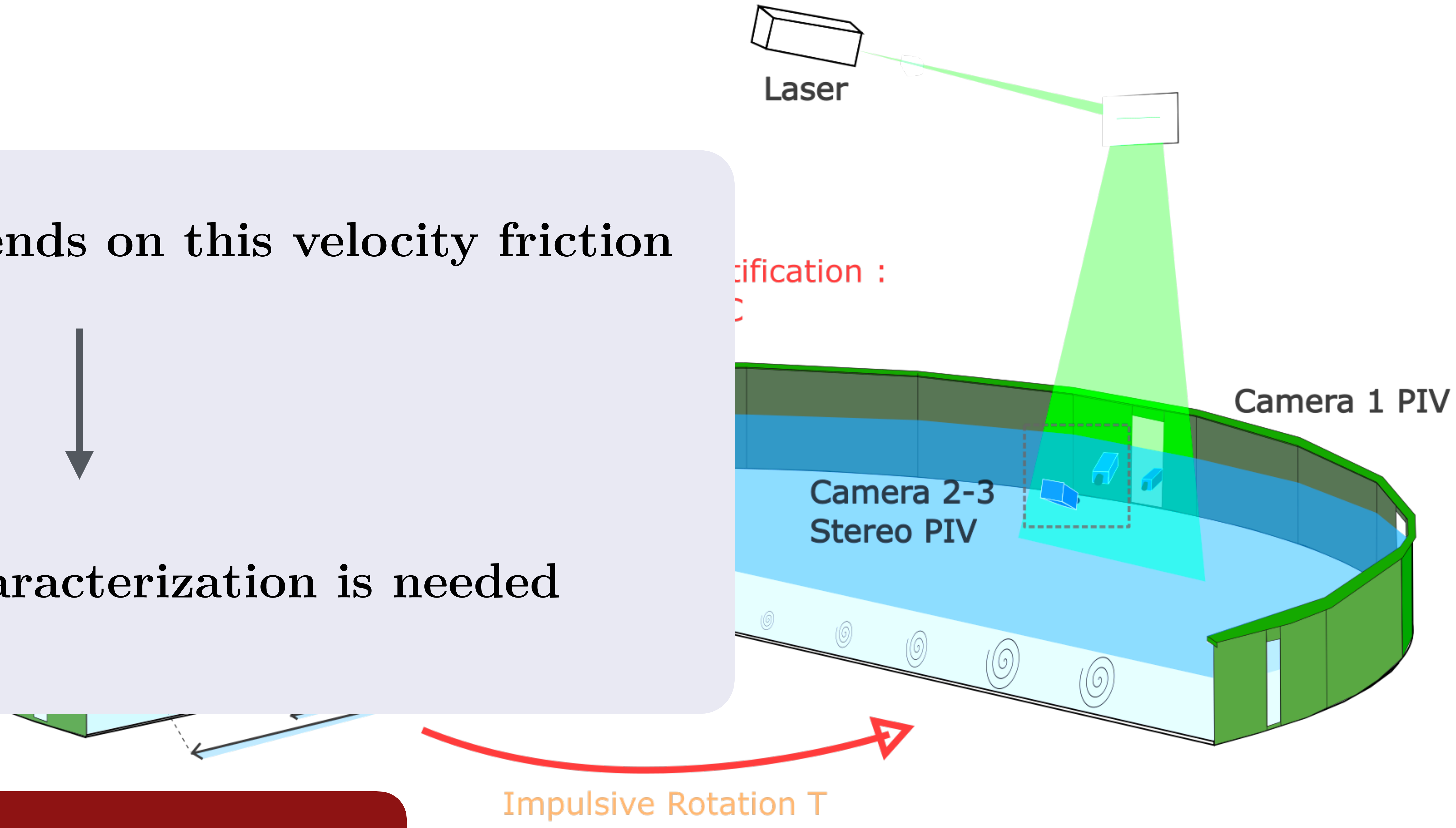
All Scalings depends on this velocity friction

↓

A proper characterization is needed



Control parameters

Friction : $u_* \sim 2\pi R\Omega/20$ **Rotation :** f **Stratification** $N^2 \equiv (\Delta T)$

Forced Convection Experiment: Vertical profiles of velocity

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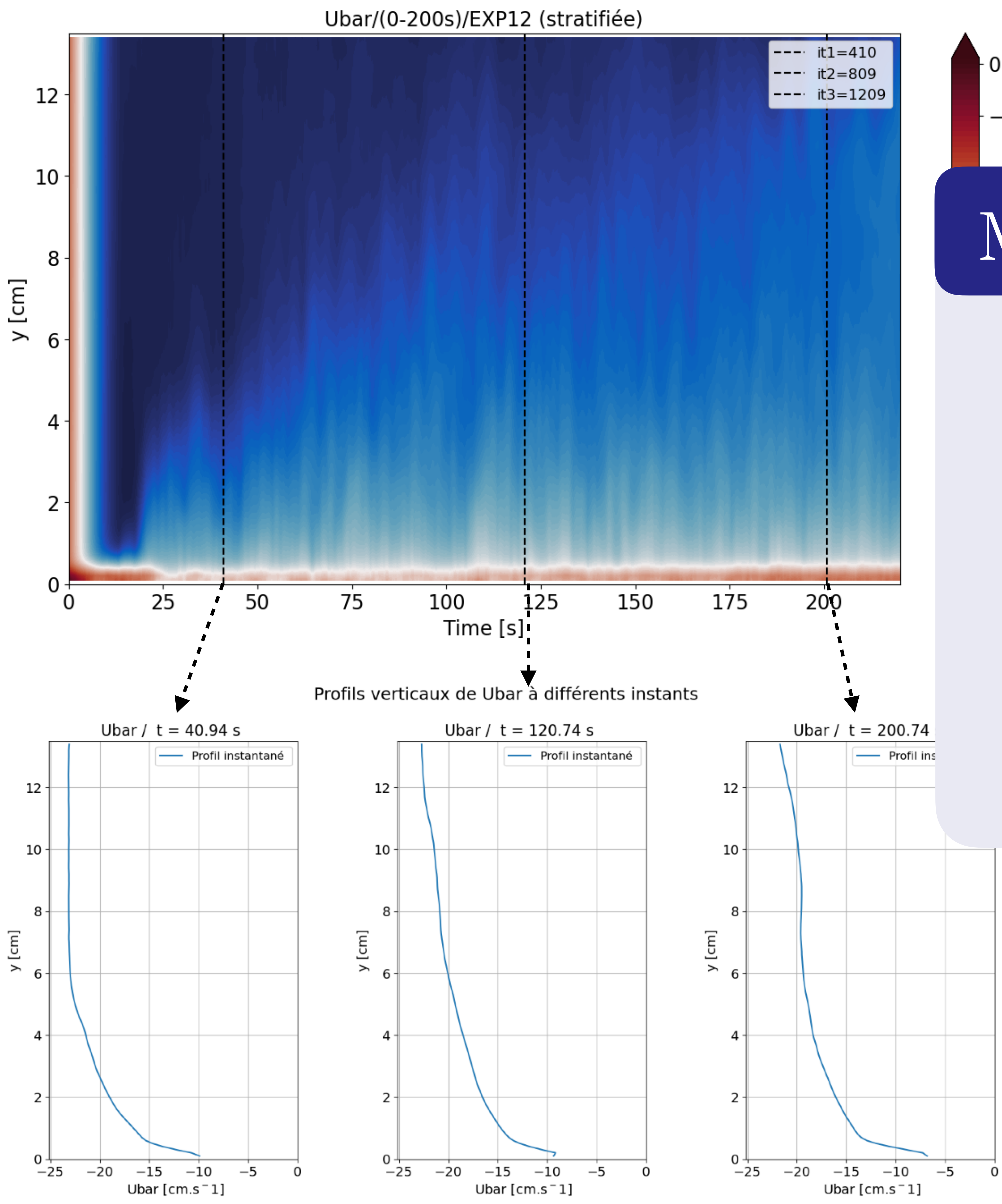

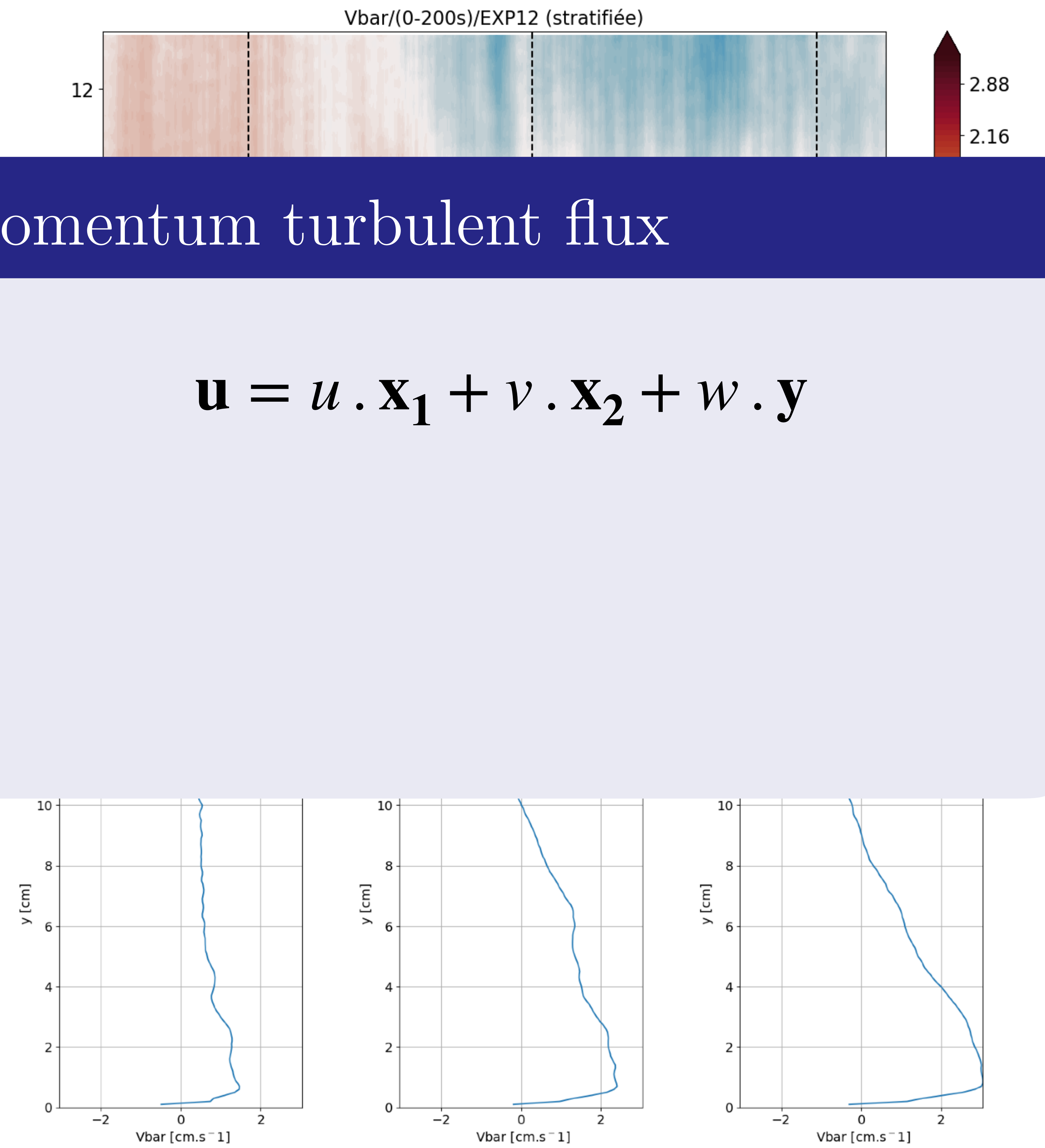

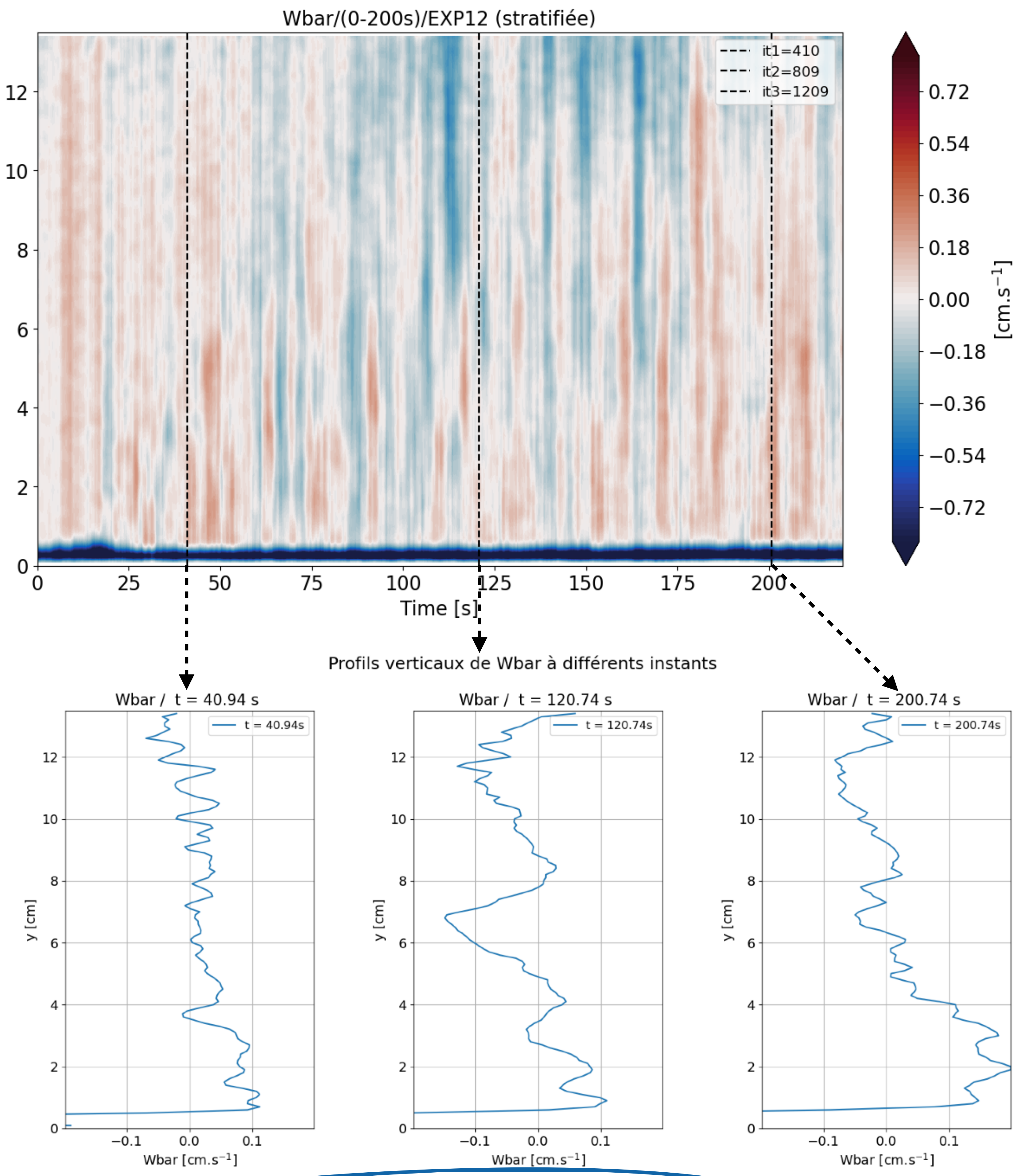

Momentum turbulent flux

$$\mathbf{u} = u\,.\,\mathbf{x_1} + v\,.\,\mathbf{x_2} + w\,.\,\mathbf{y}$$

Forced Convection Experiment: Vertical profiles of velocity

Momentum turbulent flux

$$\mathbf{u} = u\,.\mathbf{x_1} + v\,.\mathbf{x_2} + w\,.\mathbf{y}$$

$$u = \overline{u} + u' \qquad v = \overline{v} + v' \qquad w = \overline{w} + w'$$

Forced Convection Experiment: Vertical profiles of velocity

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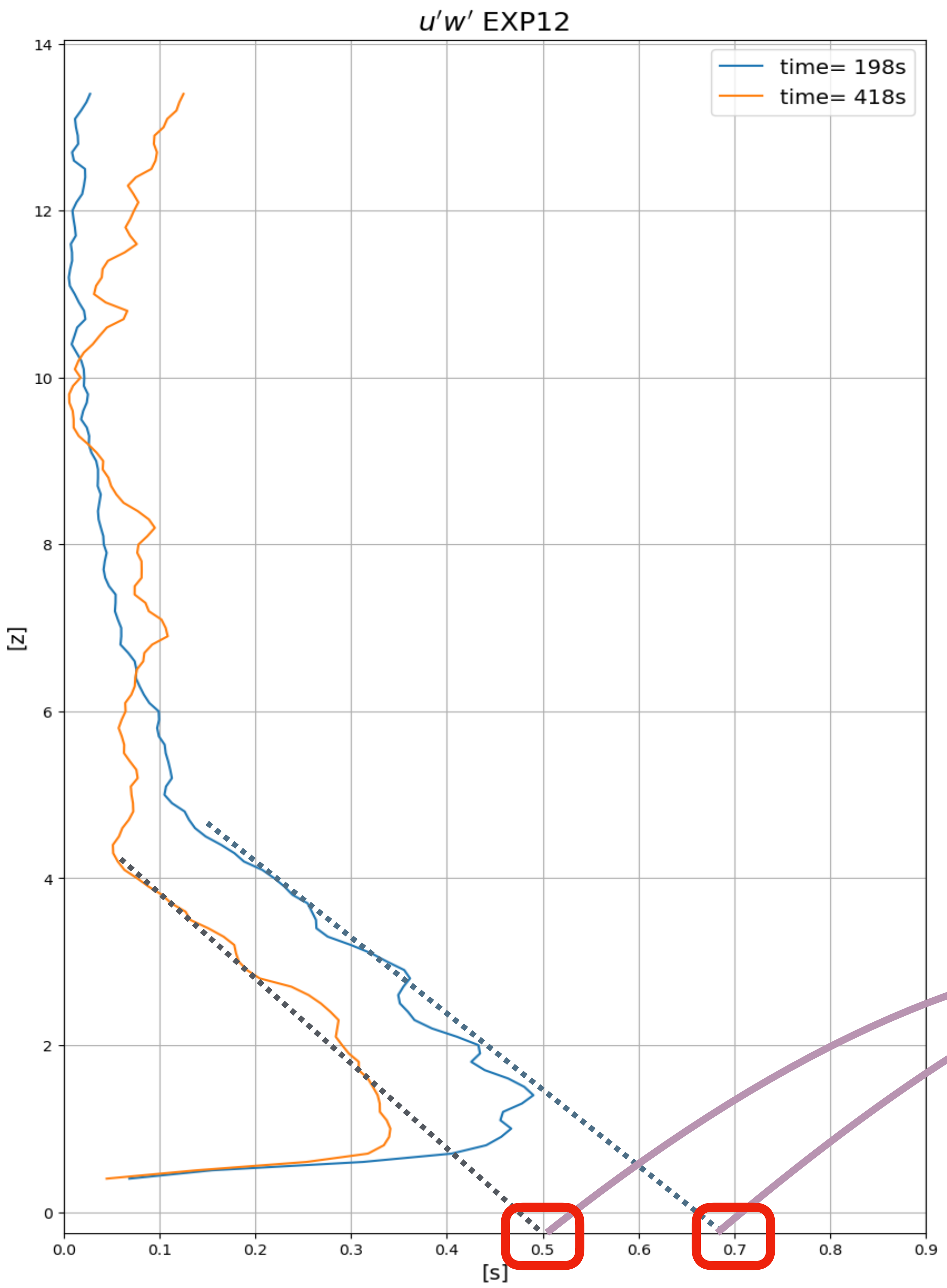
$$\overline{u'w'} \qquad \overline{v'w'}$$

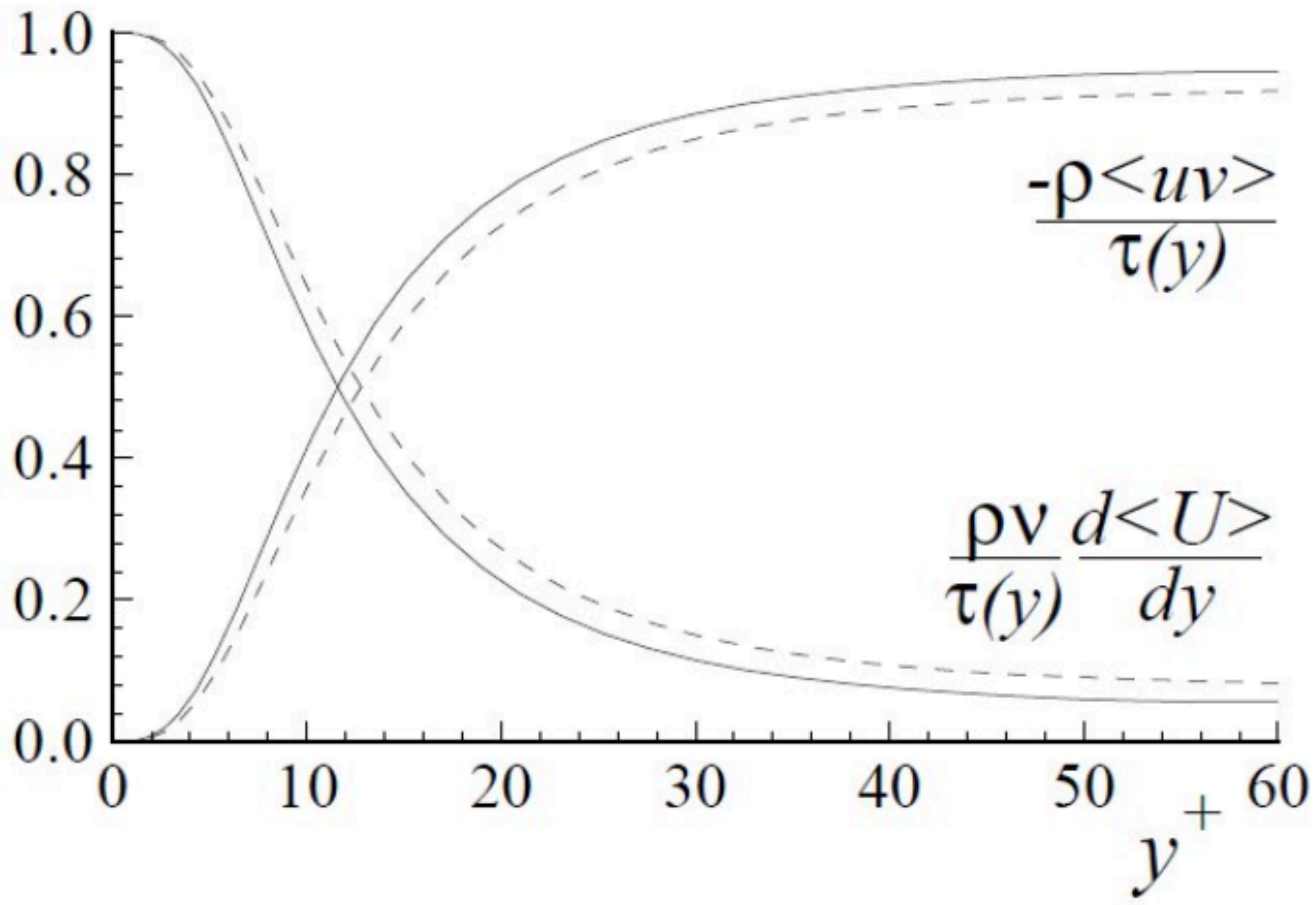
Forced Convection Experiment: Characterization of friction



Friction velocity

$$-u_*^2 = \overline{u'w'} - \nu \frac{\partial \overline{u}}{\partial z}$$

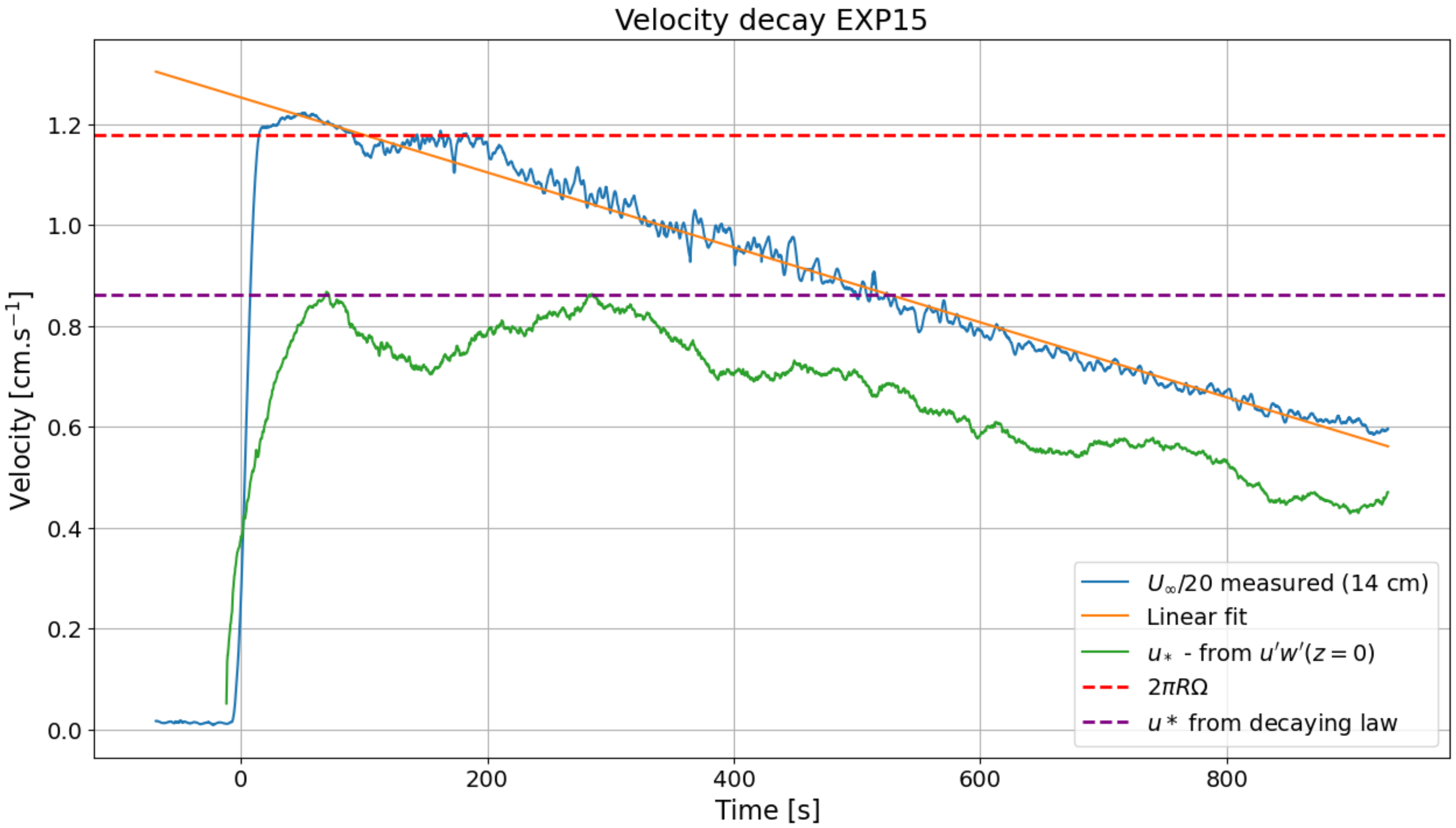
$$u_*^2(t)$$



Shear stress: turbulence + viscosity

Near the wall, the viscous and the turbulent stress sum up to a constant value (Pope, 2000)

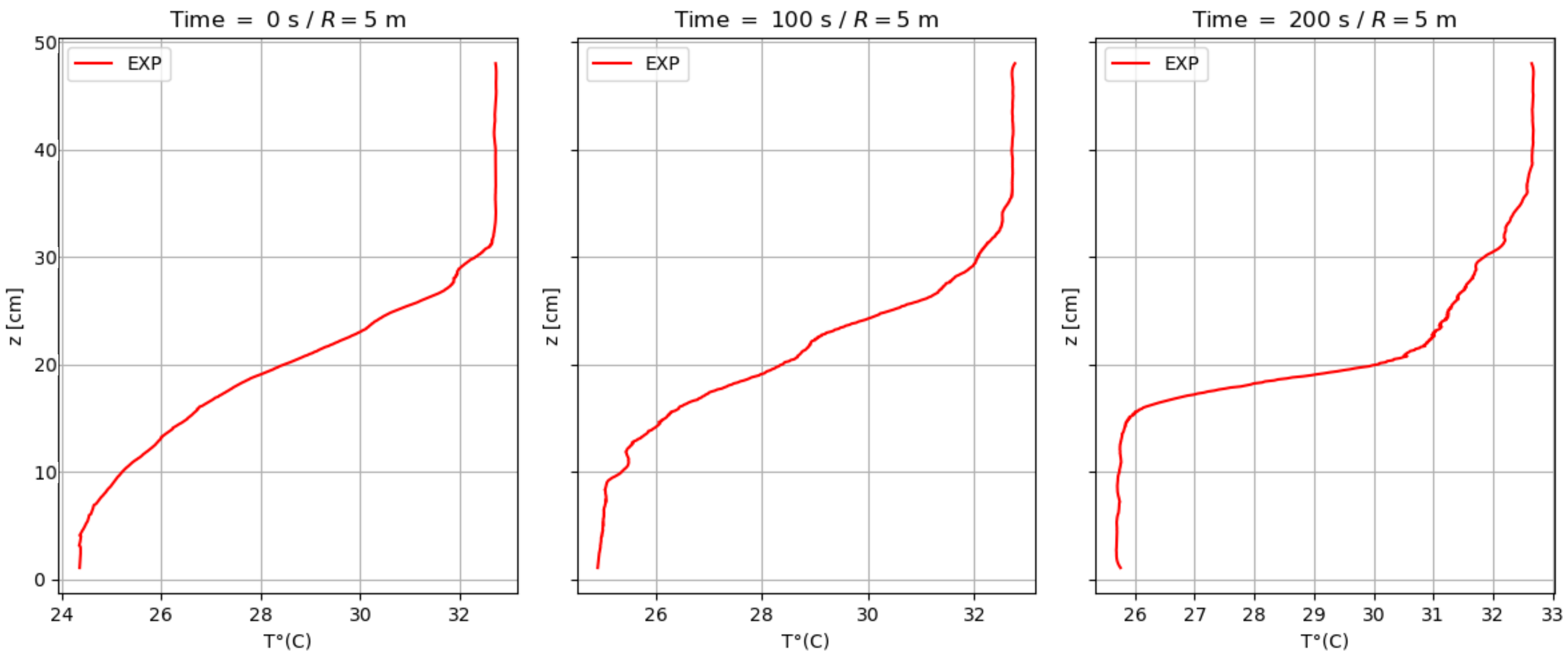
Forced convection experiment: Characterization of friction



Conservation of angular momentum

$$H\frac{dU}{dt} = -u_*^2$$

Forced Convection Experiment: Different Boundary layers



Boundary layer based on

Thermodynamical quantities

$$N^2(h_T) = \max\left(N^2\right)$$

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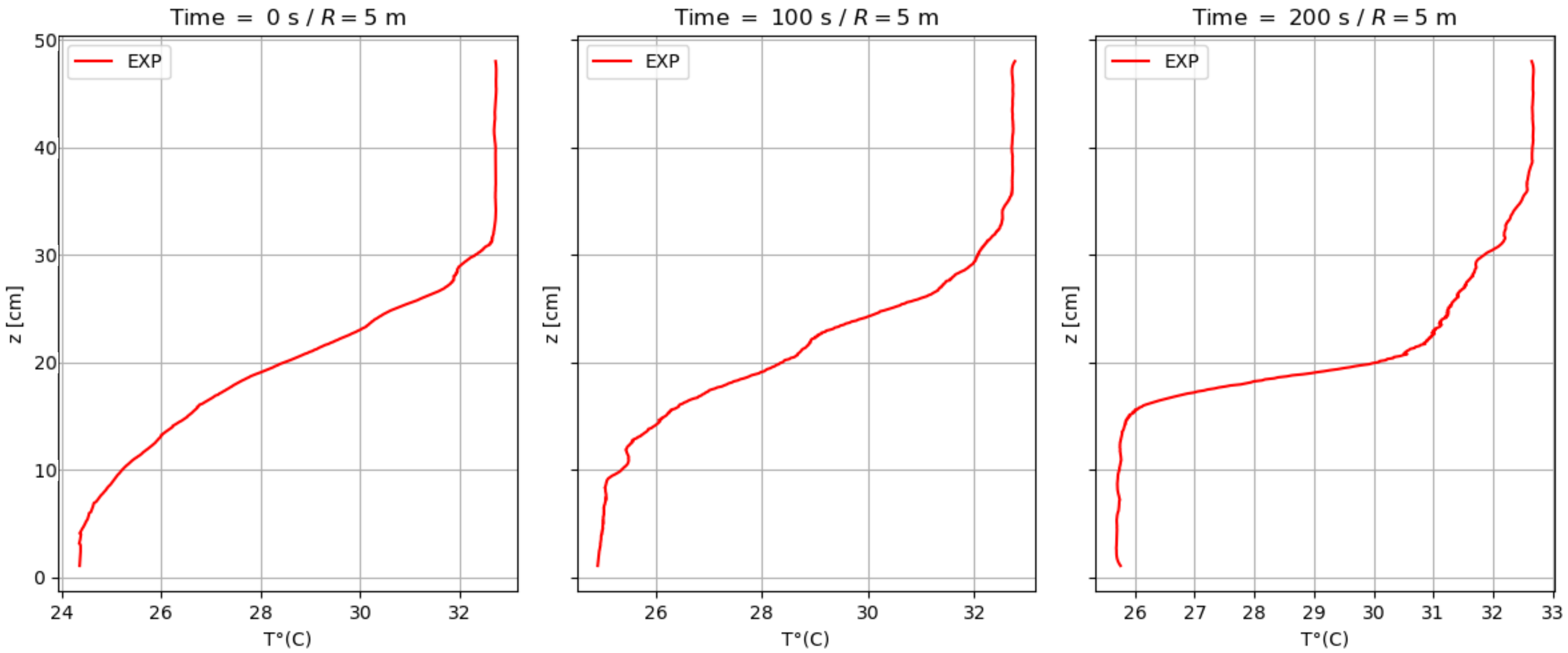

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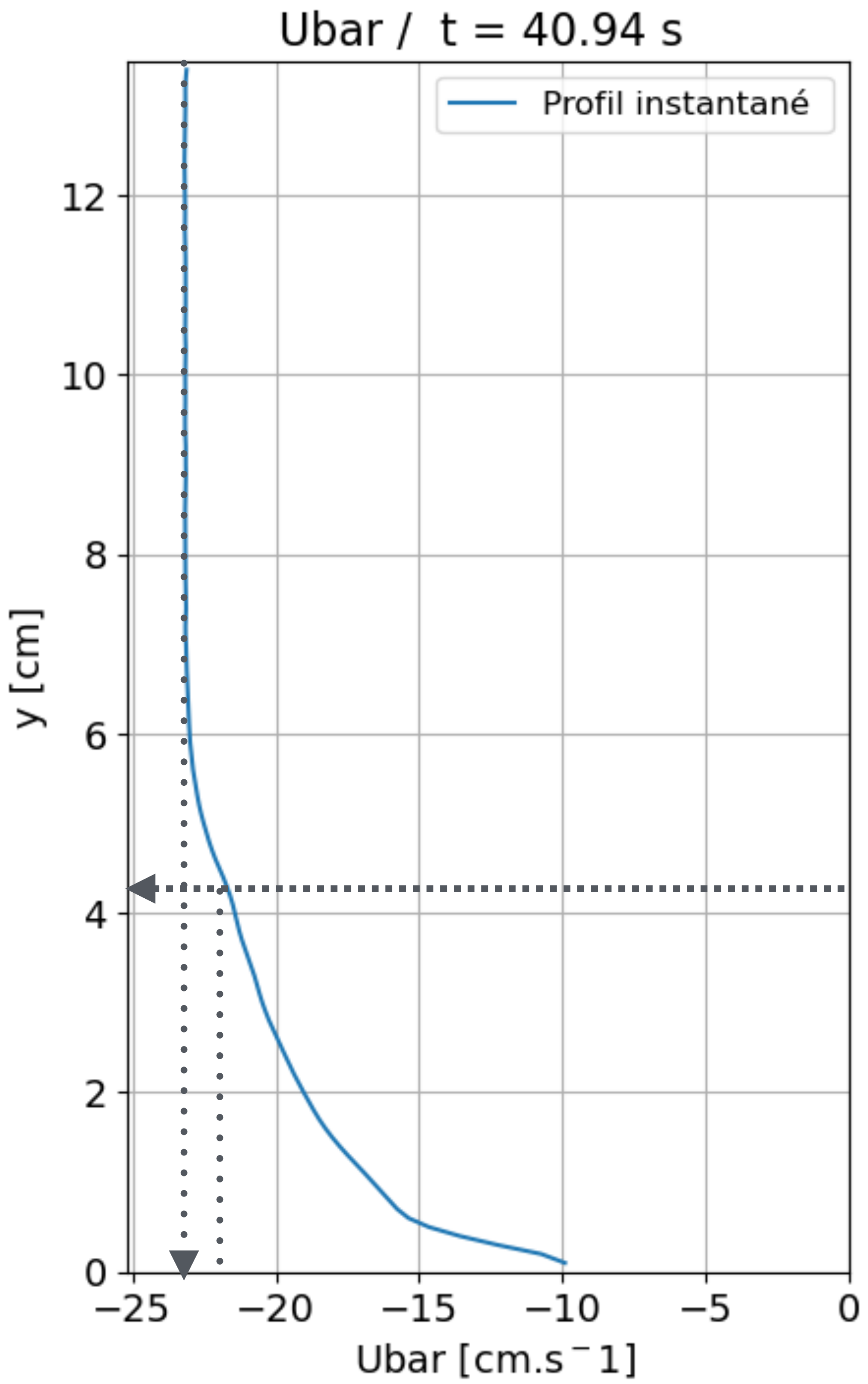
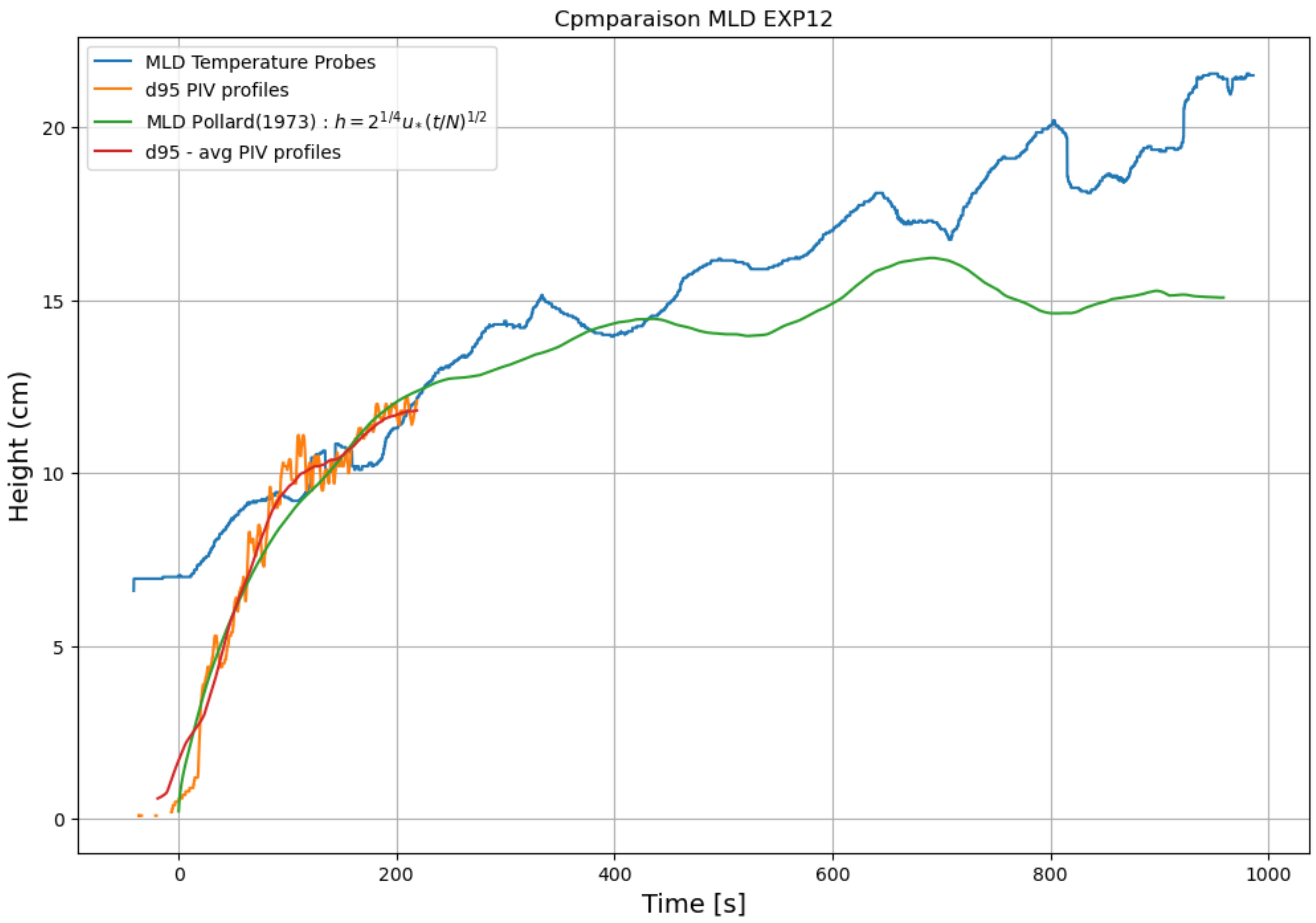


Boundary layer based on dynamical quantities

$$U(\delta_{95}(t)) = 95\,\%\,U_\infty$$

With U_∞ the velocity of the fluid in the reference frame of the plate far from the wall

Forced Convection Experiment: Growth of the mixed layer



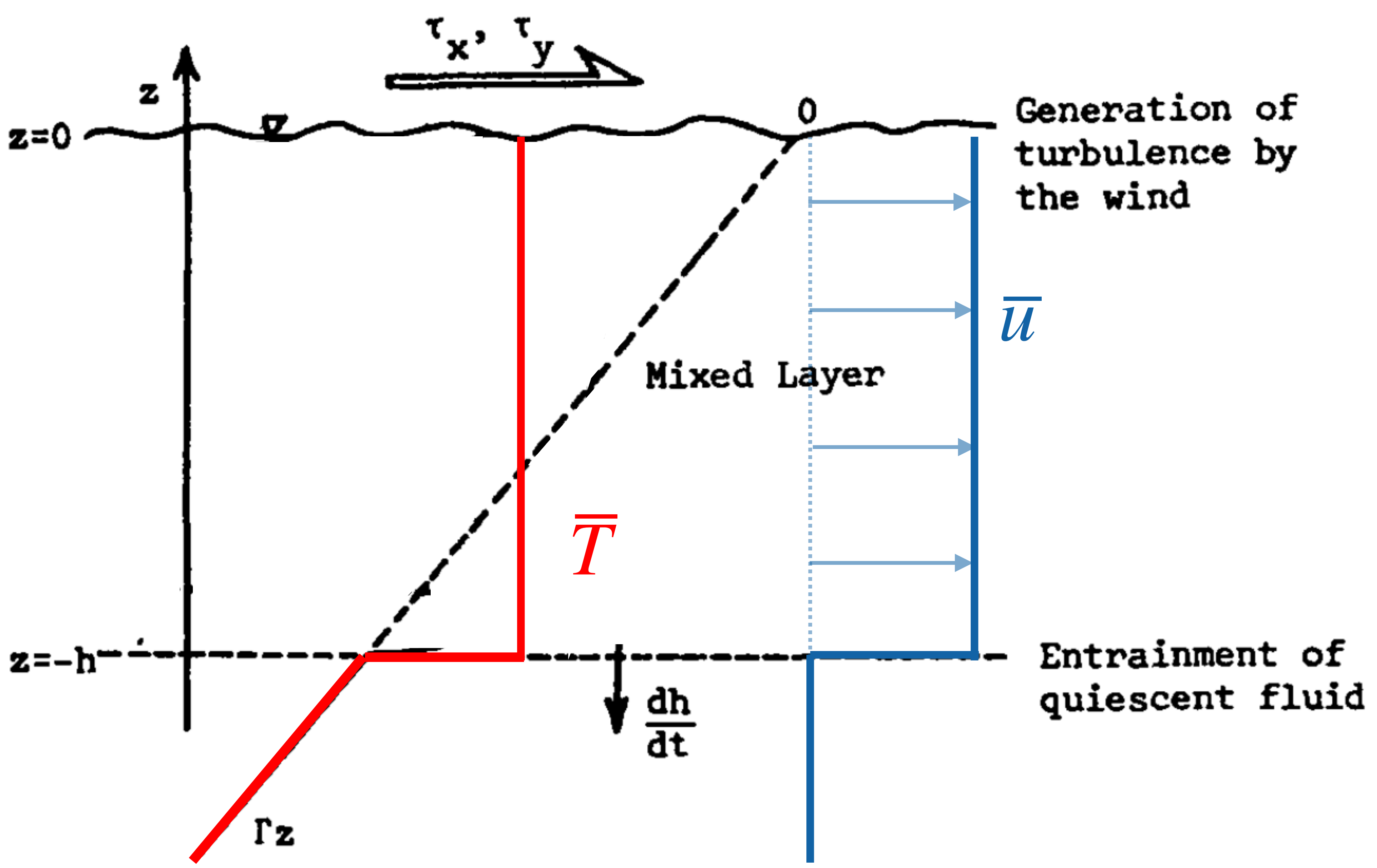
Growth of the mixed layer by momentum turbulent flux

- Accordance at short time with Pollard et al, 1973

$$h_{(t)} = 2^{1/4} u_* \left(\frac{t}{N}\right)^{1/2}$$

- Consistency between h_T and δ_{95} at short time
- At longer time $\longrightarrow$ different dynamics
  - Spin-up
  - Ekman dynamic

Longer time behavior of the ML: Slab model (Pollard et al, 1973)



- Uniform moving layer:

$$\frac{d\langle u\rangle}{dt} - f\langle v\rangle = u_*^2$$

$$\frac{d\langle v\rangle}{dt} + f\langle u\rangle = 0.$$

$$\langle u\rangle = \frac{u_*^2}{f}\sin(ft)$$

$$\langle v\rangle = -\frac{u_*^2}{f}(1-\cos(ft))$$

$$E_{slab} = \left[\langle u\rangle^2 + \langle v\rangle^2\right]/2 \quad = \frac{u_*^4}{f^2}(1-\cos ft)$$

$$\frac{dE_{slab}}{dt} = u_*^2\,\langle u\rangle$$

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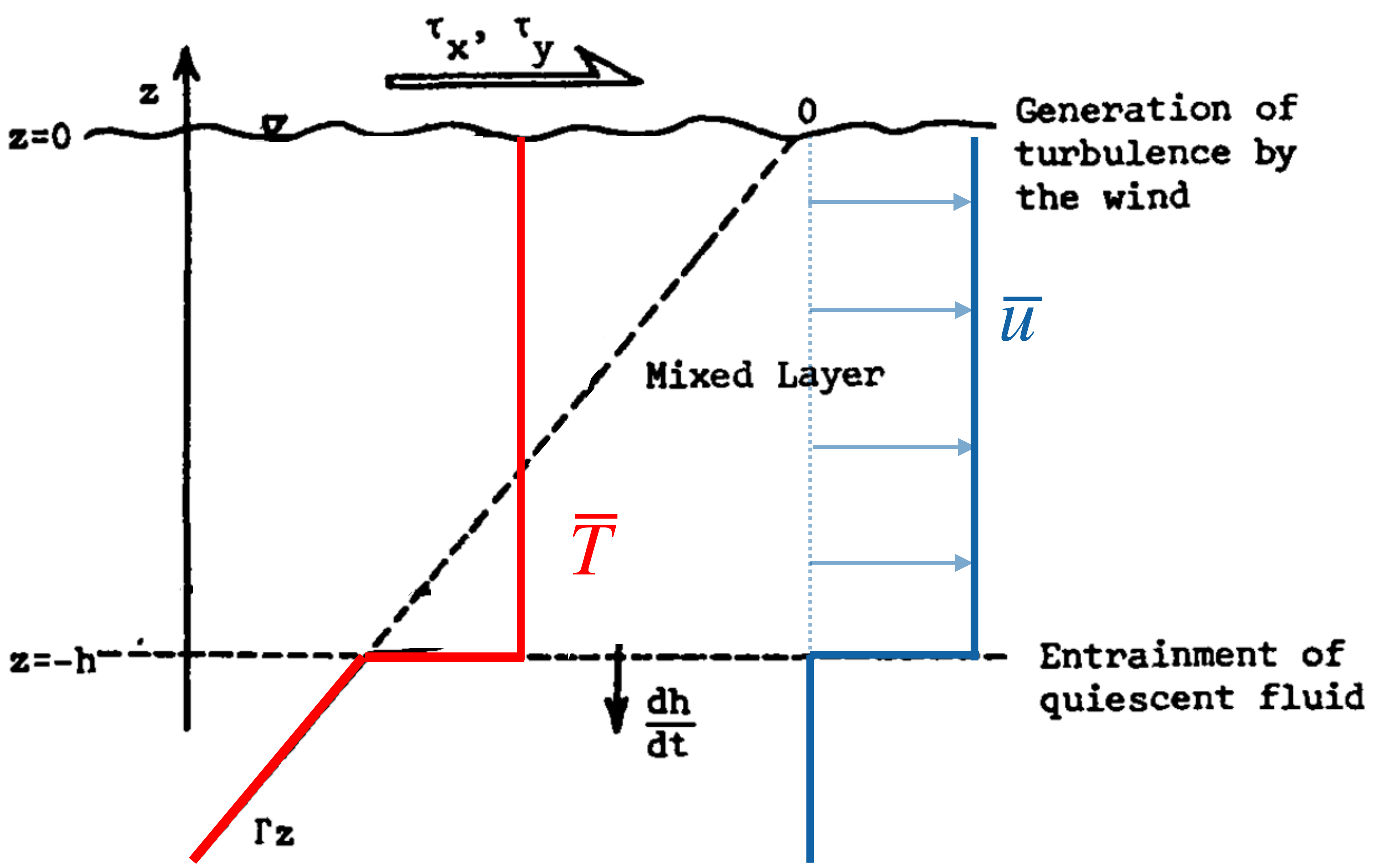

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- Potential energy: (from geometrical considerations)

$$E_{pot} = -\int_{-H}^{0} bzdz = -\int_{-H}^{-h} N_0^2 z^2 dz + N_0^2\frac{h}{2}\int_{-h}^{0} zdz = N_0^2[\frac{h^3-H^3}{3} - \frac{h^3}{4}]$$

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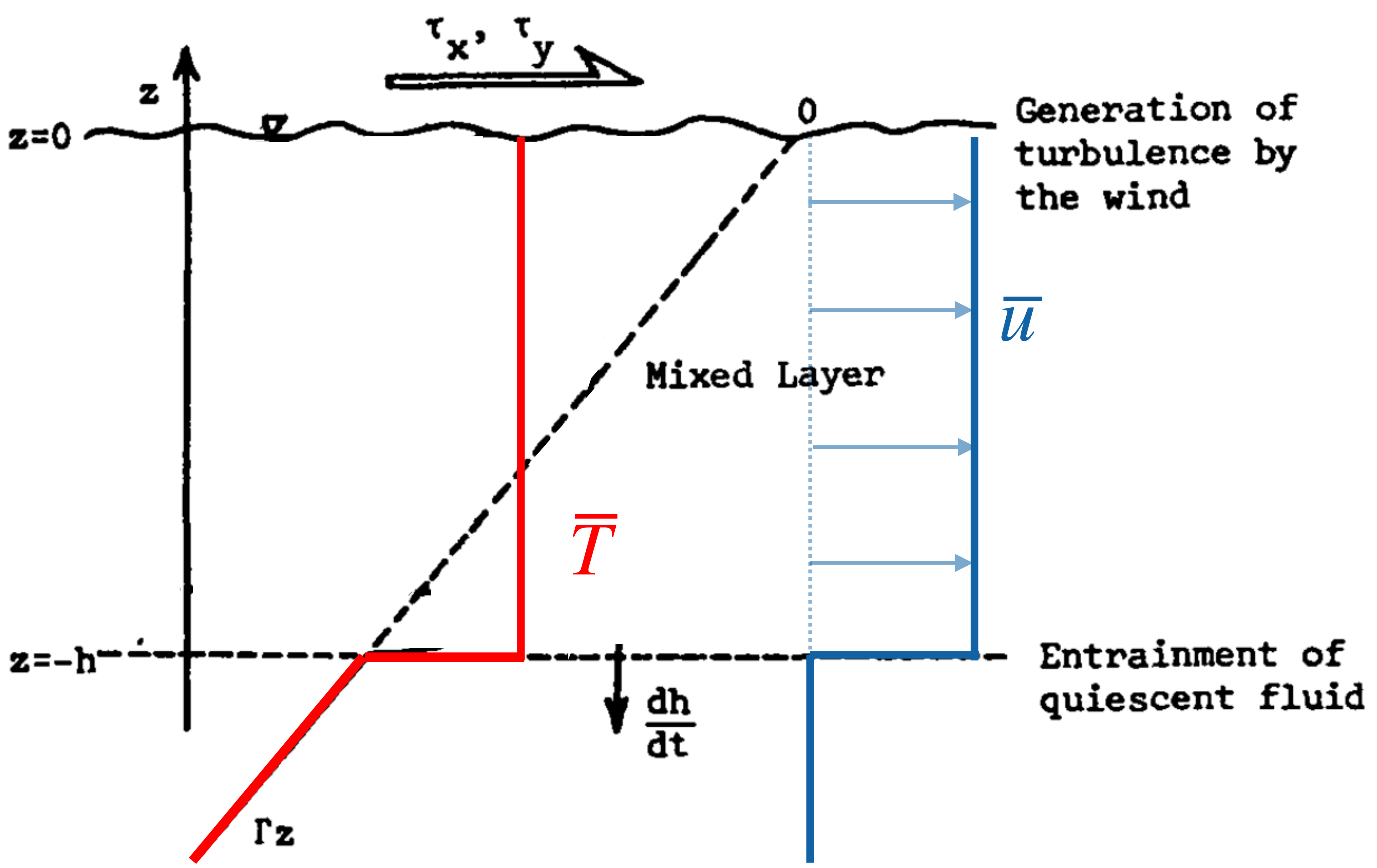

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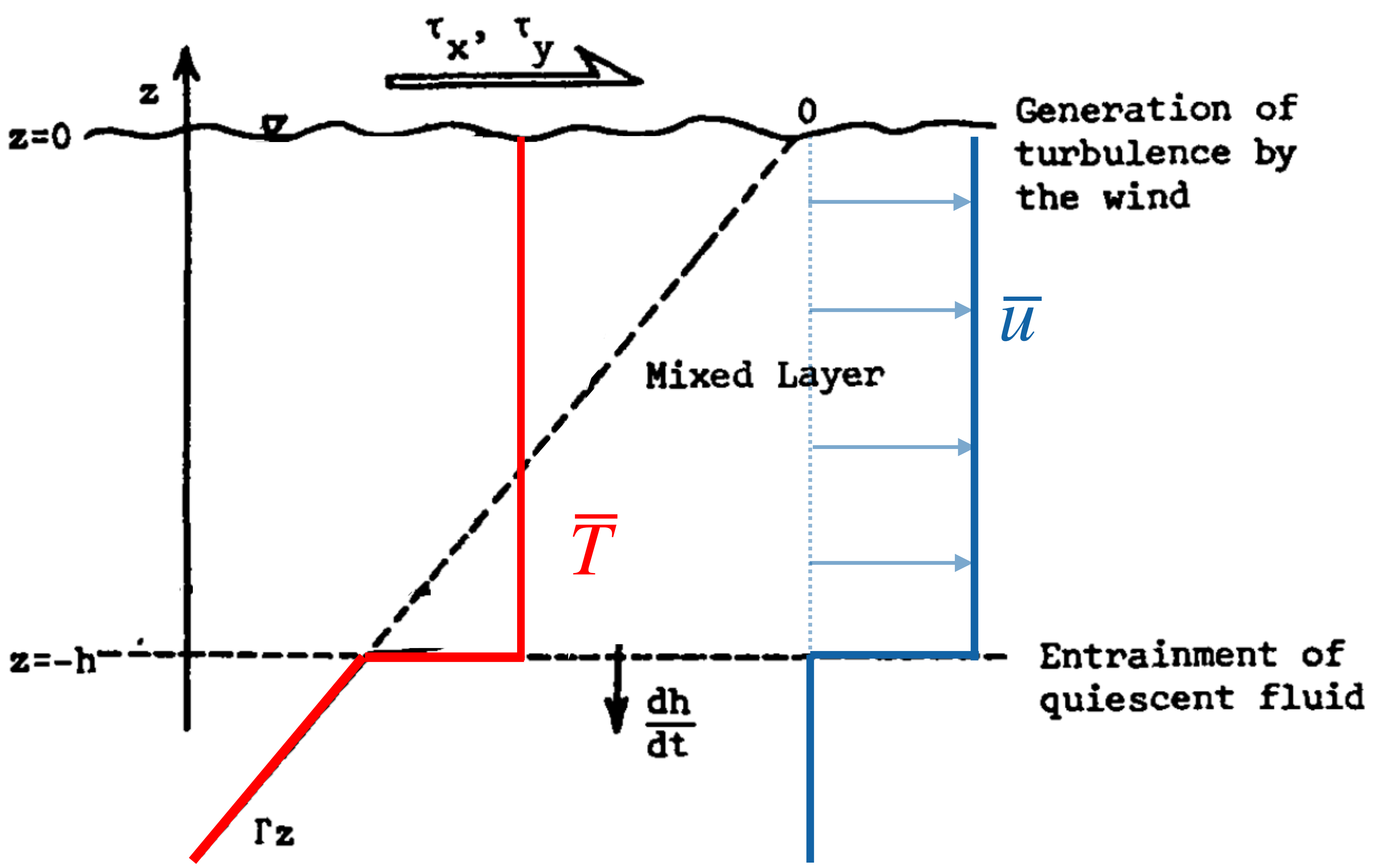
- Kinetic energy: (Since $E_{kin} = E_{slab}/h$)

$$\frac{dE_{kin}}{dt} = \frac{dE_{slab}}{h\,dt} - \frac{dh}{h^2\,dt}E_{slab}$$

Longer time behavior of the ML: Slab model (Pollard et al, 1973)



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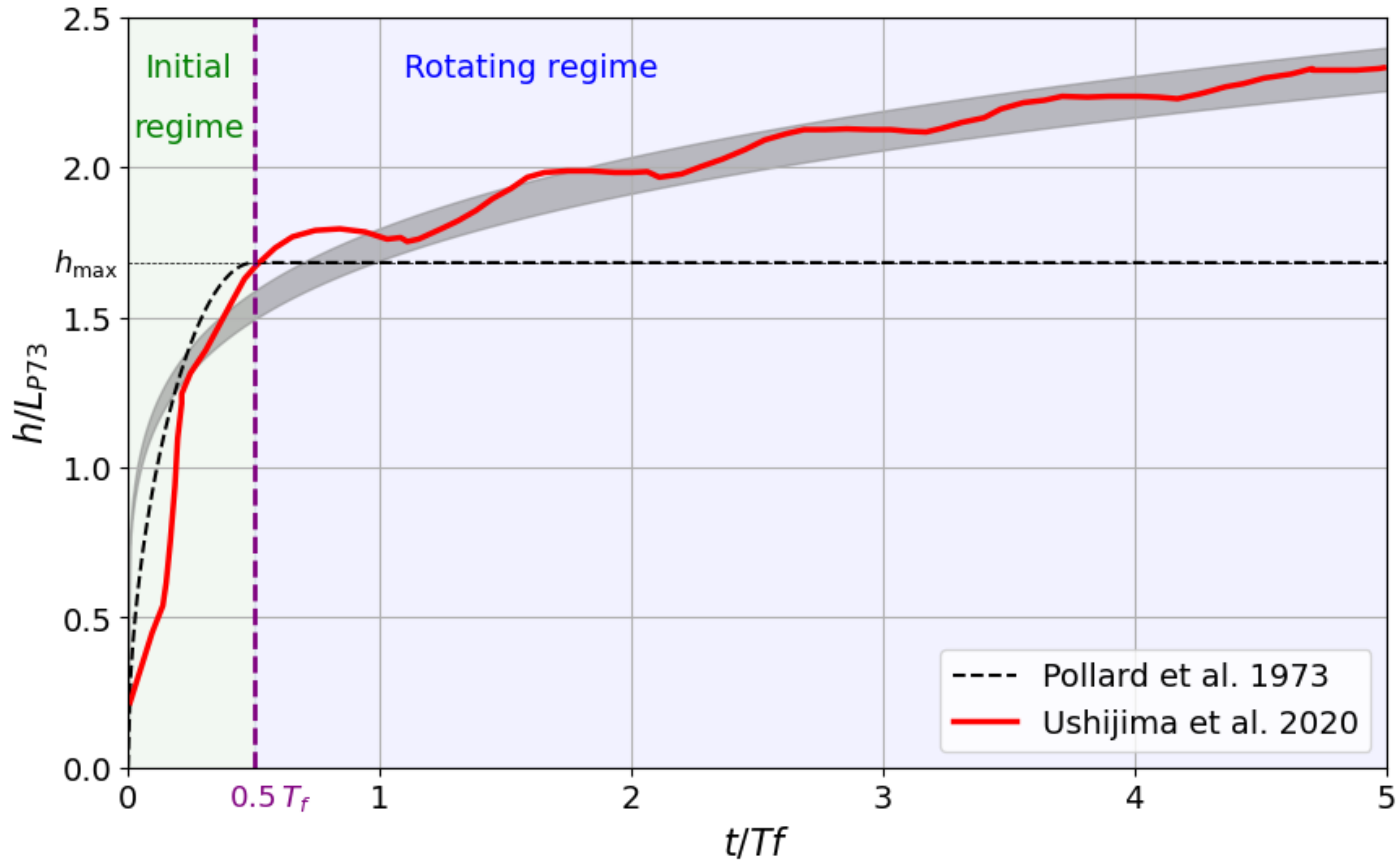
- Equation for energy:

$$\frac{dE_{kin}}{dt} + \frac{dE_{pot}}{dt} = u_*^2\ \langle u\rangle/h \quad \longrightarrow \quad -E_{slab}\frac{dh}{h^2dt} + \frac{dE_{pot}}{dt} = 0$$

$$\left[-\frac{u_*^4}{h^2f^2}(1-\cos ft) + N_0^2\frac{h^2}{4}\right]\frac{dh}{dt} = 0 \quad \longrightarrow \quad \boxed{\frac{dh}{dt} = 0 \quad \text{or} \quad h^4 = \frac{4u_*^4}{f^2N_0^2}(1-\cos ft)}$$

Longer time behavior of the ML: Slab model (Pollard et al, 1973)



At longer time deepening continue

LES: $$h = 1.5 L_{p73} \left(\frac{N_0}{f}\right)^{0.022} \left(\frac{t}{T_f}\right)^{0.18}$$

Ushijima et al, 2020

Pollard et al, 1973 fail after $1/2\,T_f$

They consider at $t > \pi/f$ that

$$u_*^2 \langle u \rangle < 0$$

$$\left[-\frac{u_*^4}{h^2 f^2}(1-\cos ft) + N_0^2 \frac{h^2}{4}\right] \frac{dh}{dt} = 0$$

$$\frac{dh}{dt} = 0 \qquad \text{or} \qquad h^4 = \frac{4u_*^4}{f^2 N_0^2}(1-\cos ft)$$

Longer time behavior of the ML: Extended Slab model

- Slab energy: $$\frac{dE_{slab}}{dt} = u_*^2 \langle u \rangle$$
- Potential energy: $$E_{pot} = N_0^2 [\frac{h^3 - H^3}{3} - \frac{h^3}{4}]$$
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$\tilde{E}_{(h)}$ expresses the kinetic energy associated with the deviation of the velocity from the uniform slab velocity.

$$\tilde{E} = \frac{1}{2u_*^2} \int (\mathbf{u} - \frac{\langle \mathbf{u} \rangle}{h})^2 dz = \frac{1}{u_*^2} \left[E_{kin} - \frac{u_*^4}{f^2 h}(1 - cos(ft) \right]$$

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> 0

- Exact equation for energy:

$$\Bigg[\underbrace{-\frac{u_*^2}{h^2 f^2}(1-\cos(ft))}_{\text{SLAB}} + \underbrace{\frac{N_0^2}{u_*^2}\frac{h^2}{4}}_{\text{POTENTIAL}}\Bigg]\frac{dh}{dt} = \overbrace{\underbrace{\left(u_{(z=0,t)} - \frac{\langle u \rangle}{h}\right)}_{\text{EXTRA PROD}} - \underbrace{\frac{\mathcal{E}}{u_*^2}}_{\text{DISSIPATION}} + \underbrace{\frac{d}{dt}\tilde{E}}_{\text{DEVIATION}} + \underbrace{\frac{d}{dt}\tilde{E}_{turb}}_{\text{IMBALANCE}}}^{=\mathcal{R}}.$$

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> 0

- Residual terms: $\mathscr{R} \simeq m_p \frac{u_*^4}{hf}$

$\mathscr{R}$ is a fraction m_p of the turbulent kinetic energy production in the entrainment layer

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- Kinetic energy: $$E_{kin} = E_{slab}/h + u_*^2 \tilde{E}_{(h)}$$

$\tilde{E}_{(h)}$ expresses the kinetic energy associated with the deviation of the velocity from the uniform slab velocity.

$$\tilde{E} = \frac{1}{2u_*^2} \int (\mathbf{u} - \frac{\langle \mathbf{u} \rangle}{h})^2 dz = \frac{1}{u_*^2} \left[E_{kin} - \frac{u_*^4}{f^2 h}(1 - cos(ft) \right]$$

> 0

- Exact equation for energy:

$$\left[\underbrace{-\frac{u_*^2}{h^2 f^2}(1 - \cos(ft))}_{\text{SLAB}} + \underbrace{\frac{N_0^2}{u_*^2}\frac{h^2}{4}}_{\text{POTENTIAL}} \right] \frac{dh}{dt} = \overbrace{\underbrace{\left(u_{(z=0,t)} - \frac{\langle u \rangle}{h} \right)}_{\text{EXTRA PROD}} - \underbrace{\frac{\mathcal{E}}{u_*^2}}_{\text{DISSIPATION}} + \underbrace{\frac{d}{dt}\tilde{E}}_{\text{DEVIATION}} + \underbrace{\frac{d}{dt}\tilde{E}_{turb}}_{\text{IMBALANCE}}}^{=\mathcal{R}}.$$

- Residual terms: $$\mathscr{R} \simeq m_p \frac{u_*^4}{hf}$$

$\mathscr{R}$ is a fraction m_p of the turbulent kinetic energy production in the entrainment layer

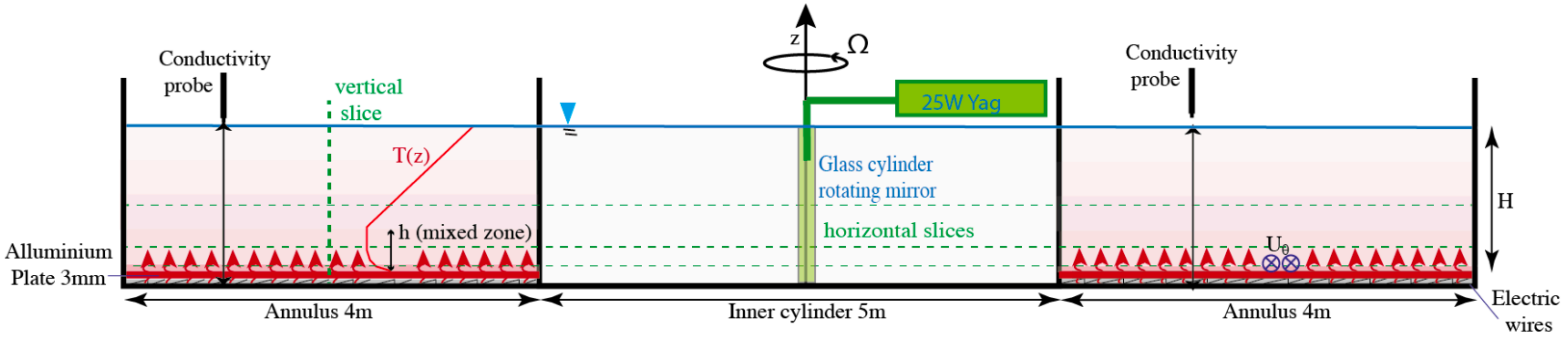
$$\left[-\frac{u_*^4}{f^2 h^2}(1 - \cos(ft)) + N_0^2 \frac{h^2}{4} \right] \frac{dh}{dt} = m_p \frac{u_*^4}{hf}$$

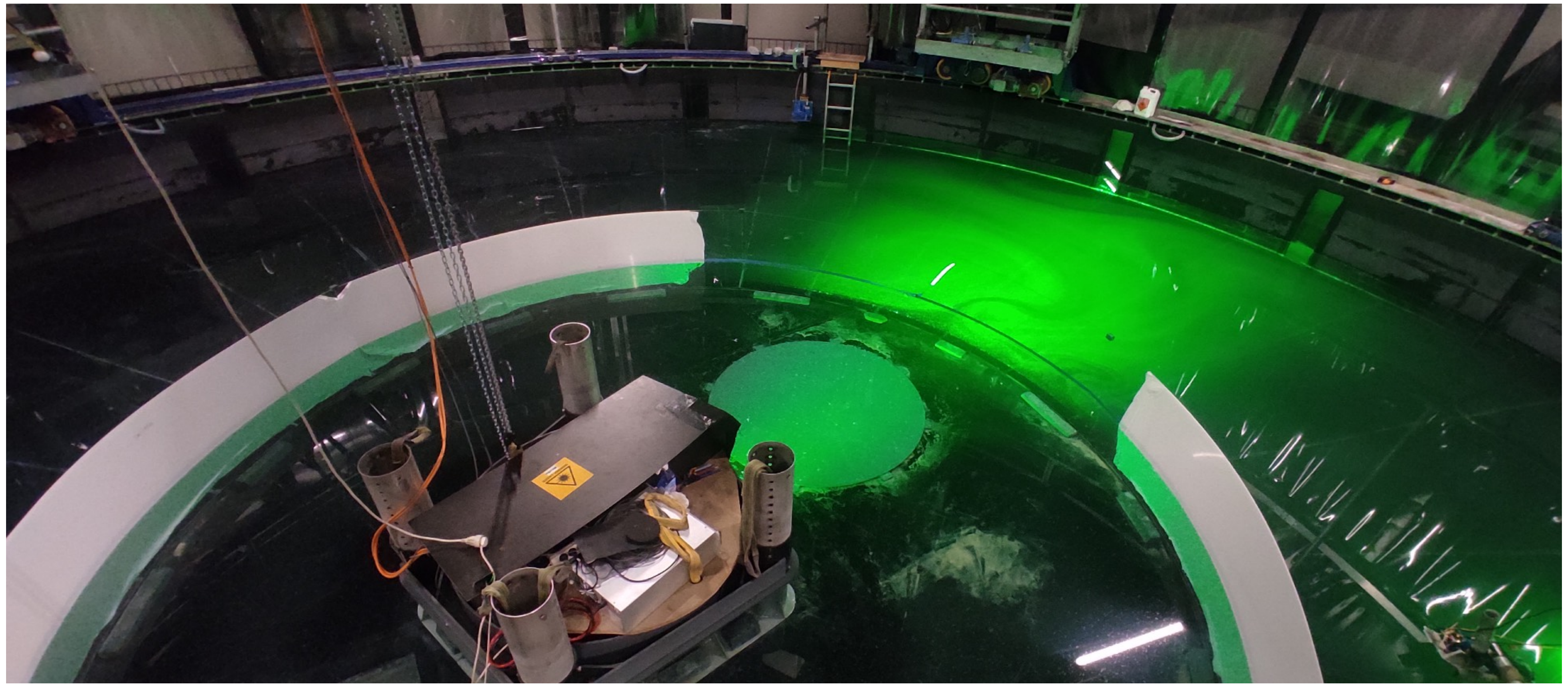
Consistant with scaling from LES

$$h^3 \frac{dh}{dt} = 4 m_p \frac{u_*^4}{N_0^2 f}$$

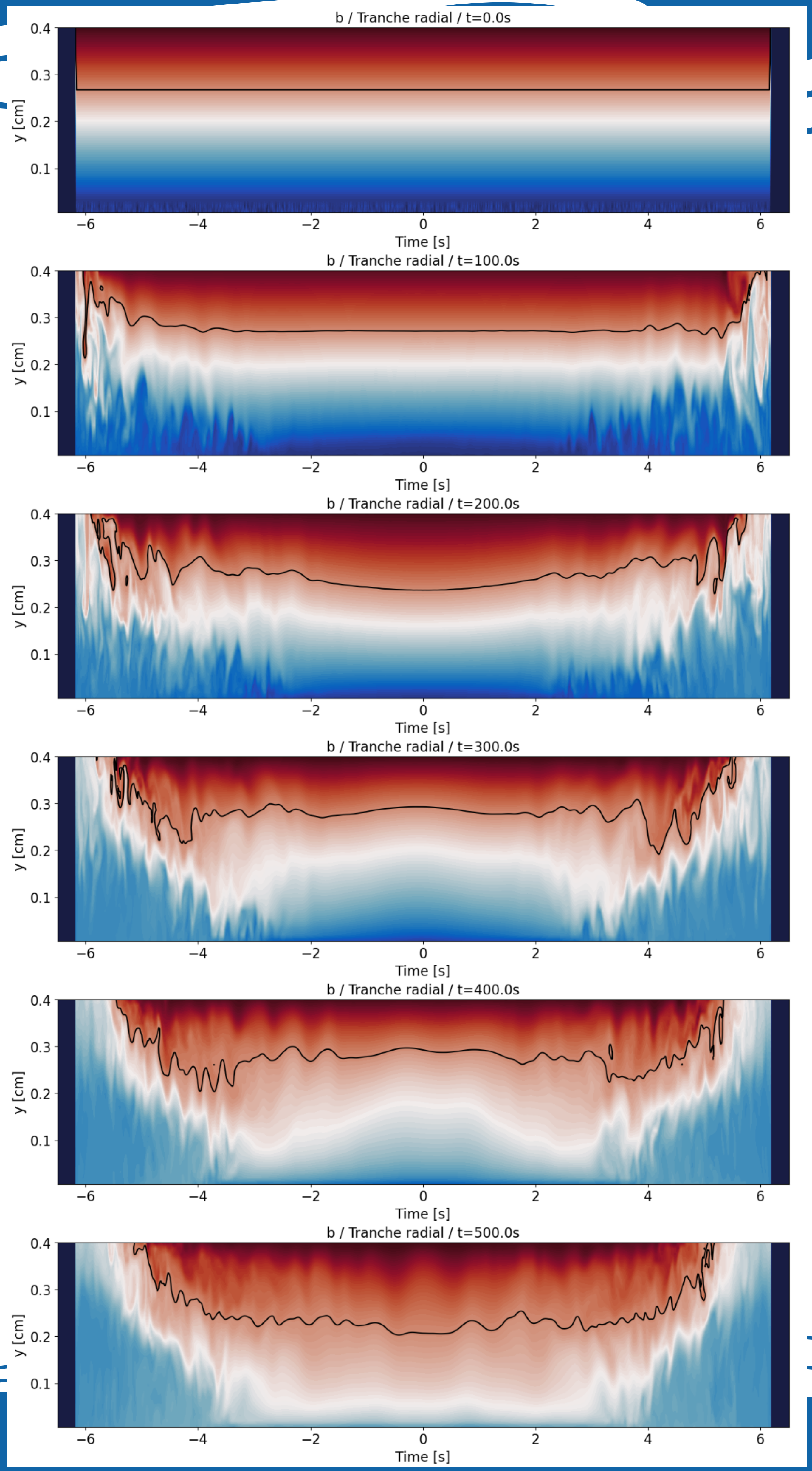
Next step : Free / Mixed Convection

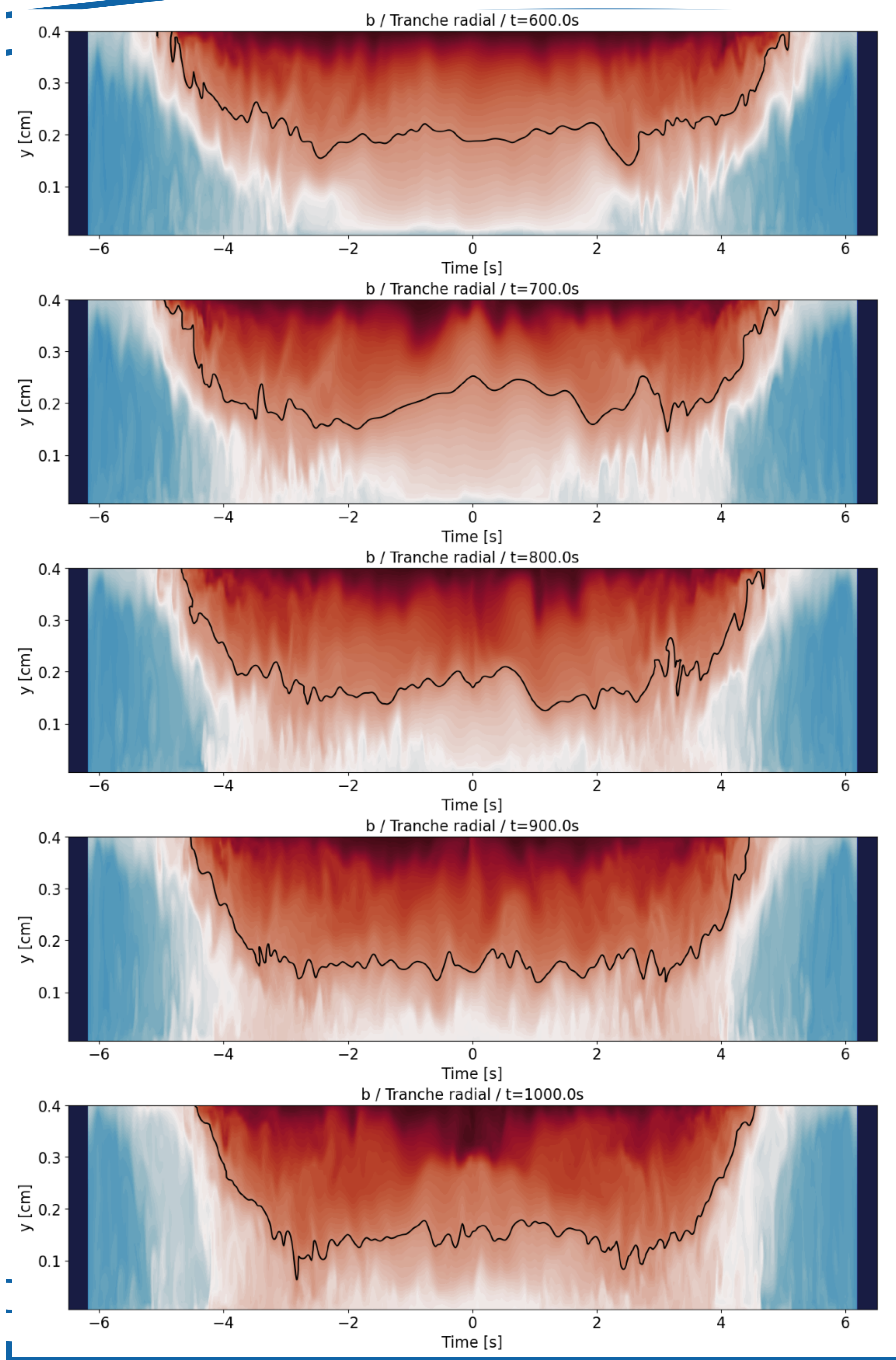
- **Heated floor [290-353] kW**
- **Inner cylinder (5m)**
- **Temperature probes**
  - 3 Vertical profilers
  - 2 Fixed probes (z =0; 12cm)
- **Vertical laser sheet (30x25)cm**
  - PIV Stereo
- **Horizontal laser sheet (3x4)m**
  - PIV (z = 10cm)
  - PIV in volume (multi- layer)
- **IR camera (3x4)m**





LES: Numerical Twins



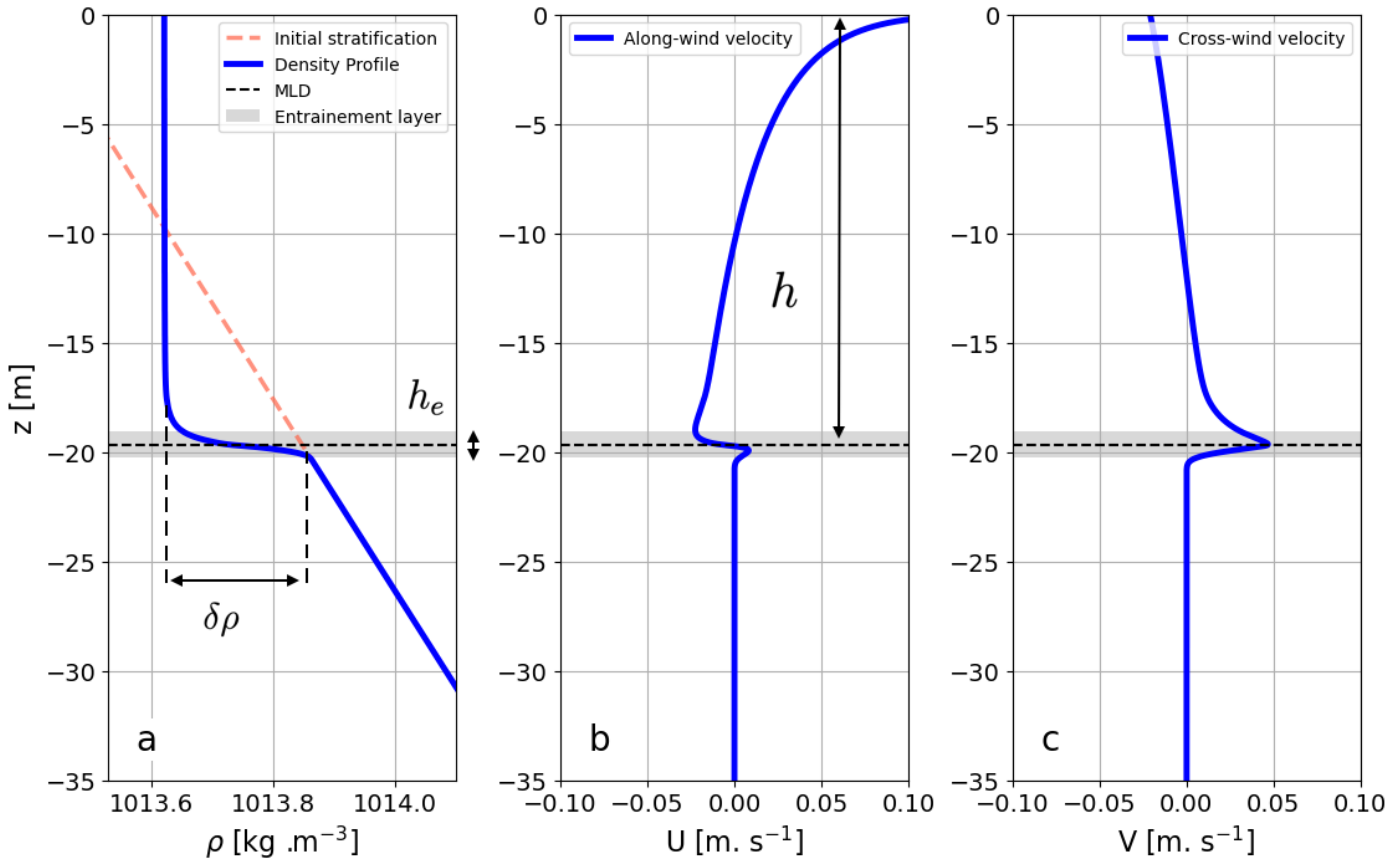


- At longer time different dynamics
  - **Strong radial effects**
  - **Wavy dynamics**

Basilics

LES - 1024 x 1024 x 32

Longer time behavior of the ML: Entrainment layer model



- Considering a fully mixed layer and an entrainment layer h_e

  Mixing occurs only in the entrainement layer → Marginal stability

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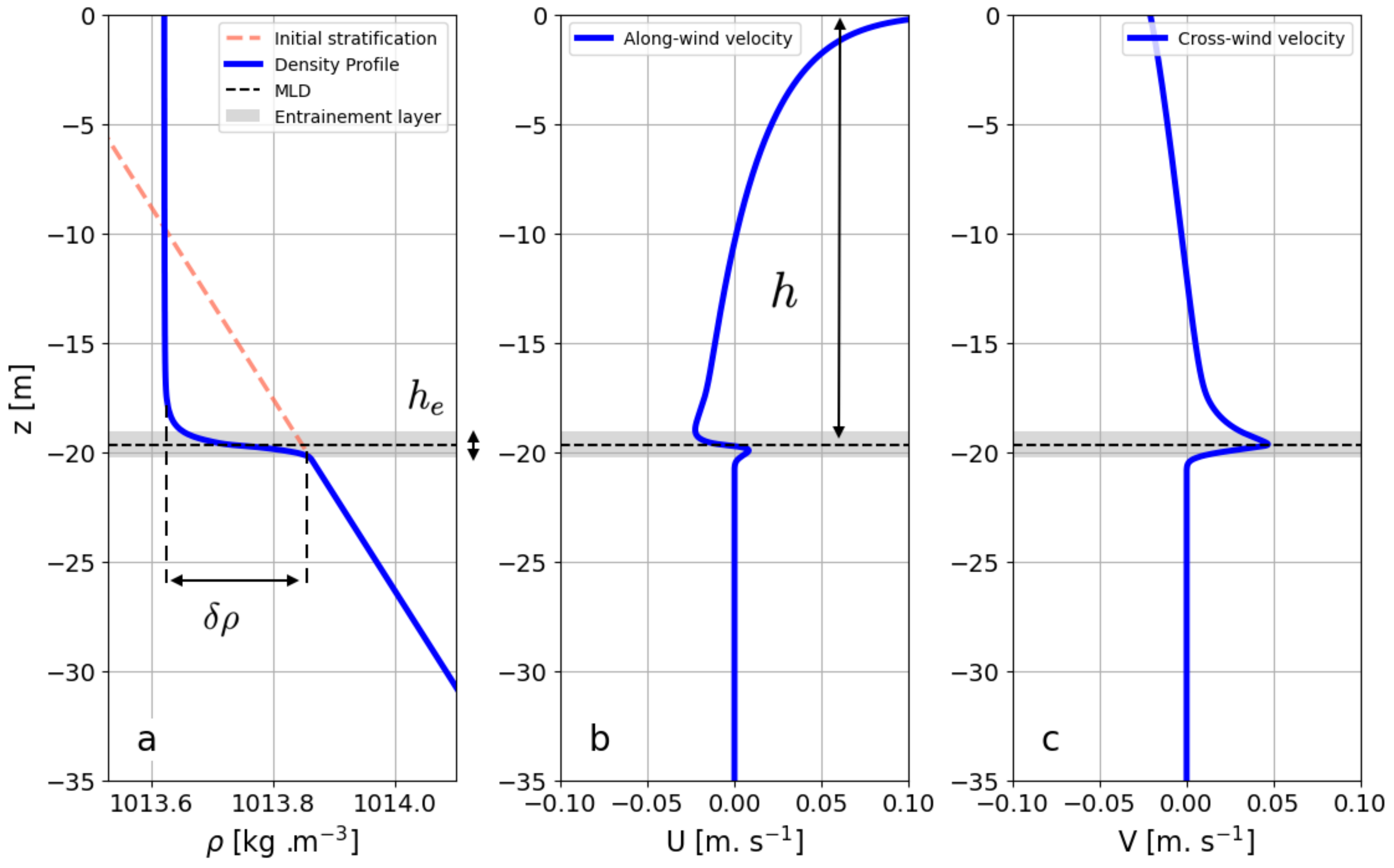

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Velocity drop (Averaged over T_f)
$$(\partial u/\partial z)^2 \simeq 2u_*^4/(f^2 h_e^2)$$

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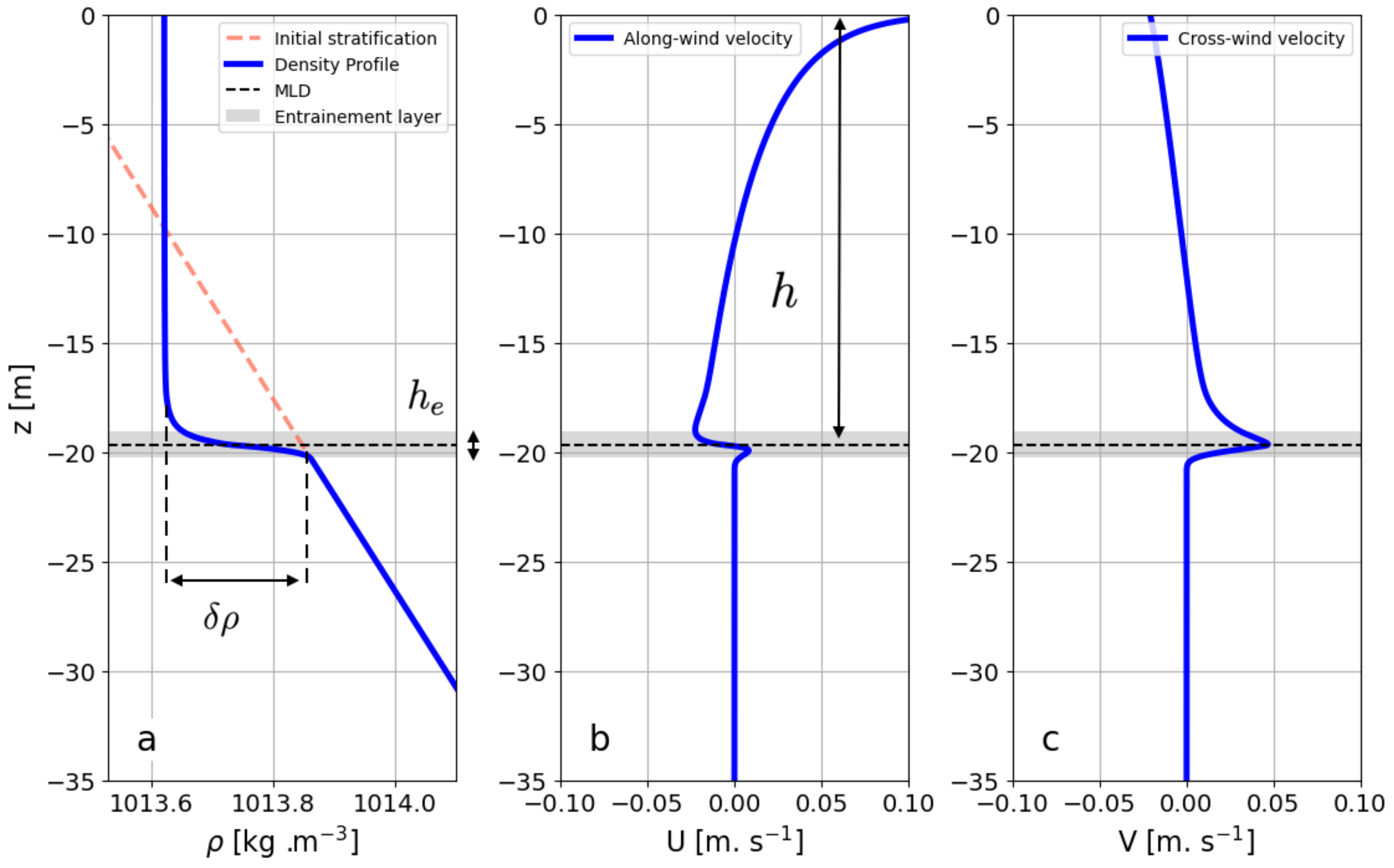

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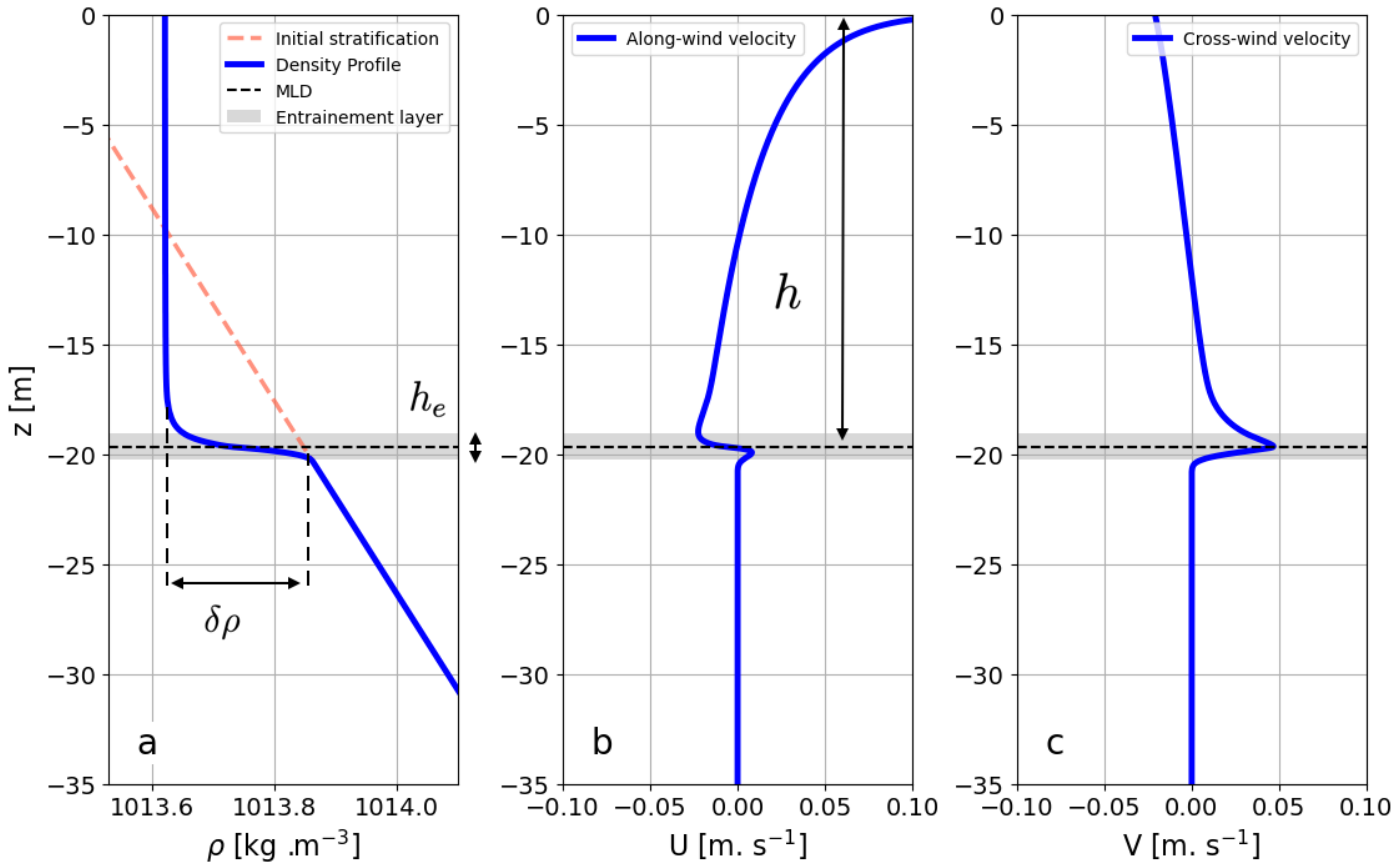

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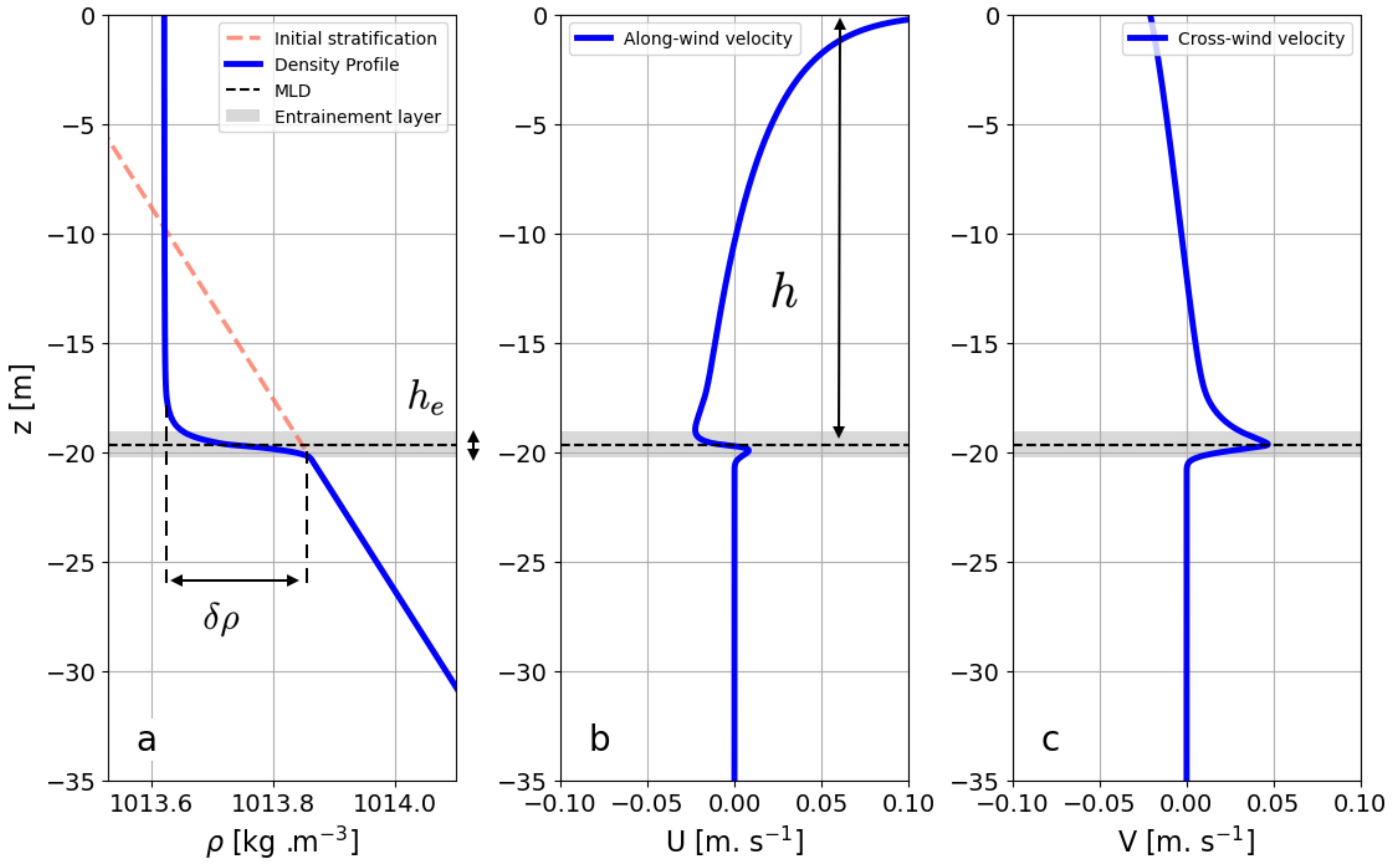

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- Turbulent Viscosity: (From dimensional argument)

$$\nu_t \simeq u_*^2/N_e \simeq 2\sqrt{2}Ri_{st}^{1/2}\frac{u_*^4}{h^2 N_0^2 f}$$

- TKE Production: (integrated over the layer) $< P >_e \simeq \nu_t u^2/h_e$

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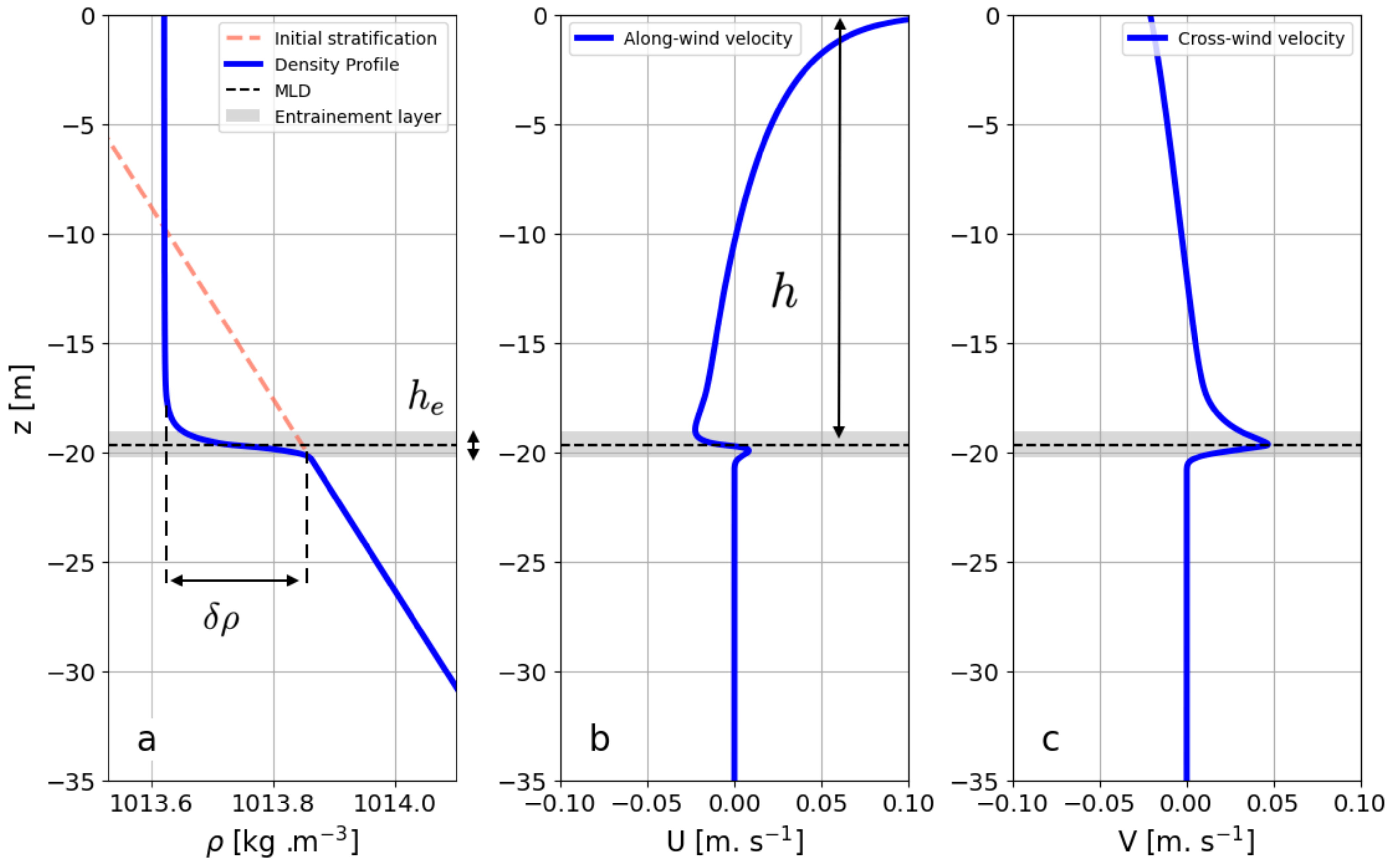
Longer time behavior of the ML: Entrainment layer model



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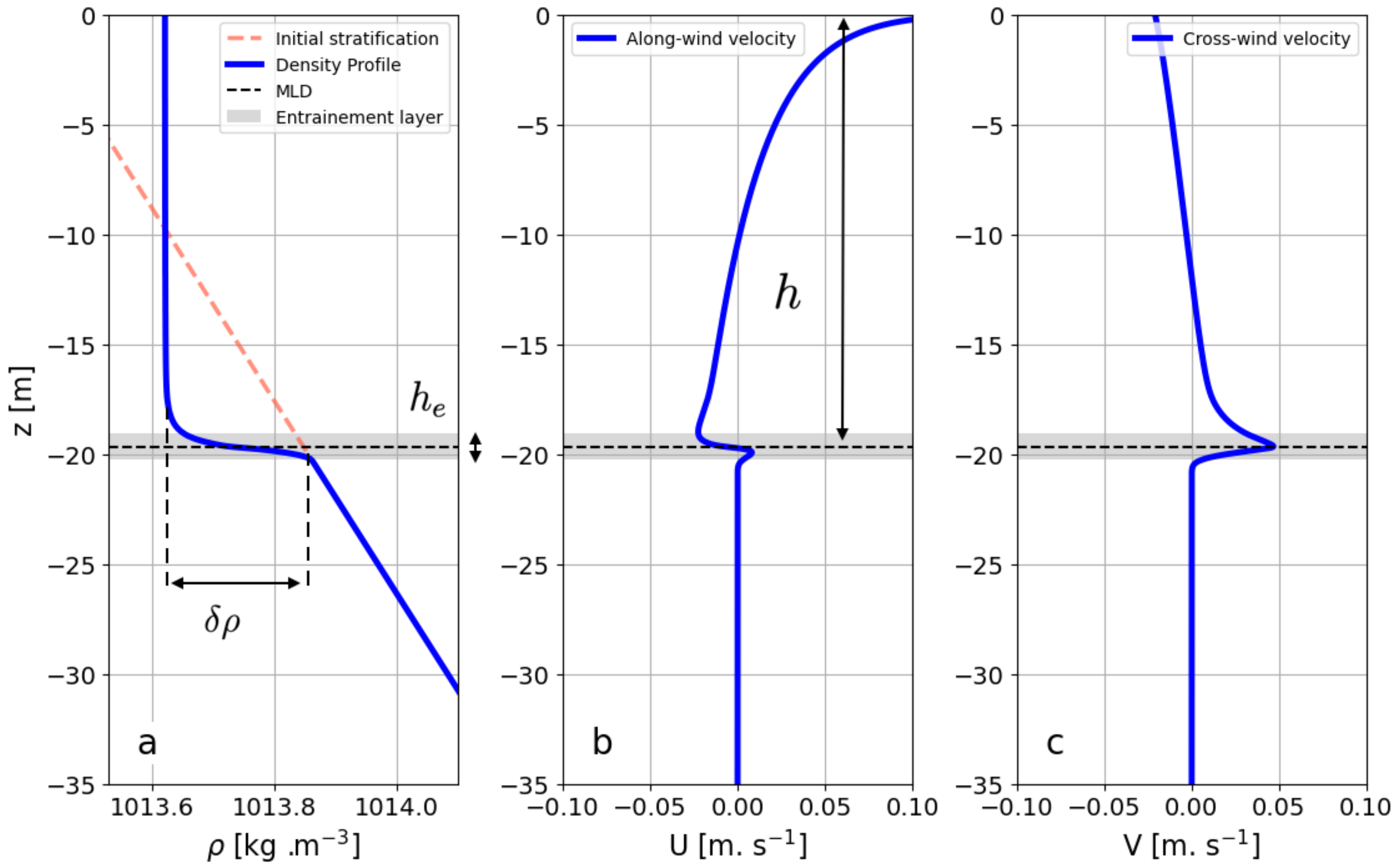

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